

Q.No.1 $\frac{x^3+2x+2}{(x^2+x+1)^2}$

Consider $\frac{x^3+2x+2}{(x^2+x+1)^2} = \frac{Ax+B}{x^2+x+1} + \frac{Cx+D}{(x^2+x+1)^2}$

ming both sides by $(x^2+x+1)^2$

$$x^3+2x+2 = (Ax+B)(x^2+x+1) + Cx+D$$

$$\Rightarrow x^3+2x+2 = A(x^3+x^2+x) + B(x^2+x+1) + Cx+D$$

Comparing co-efficients of x^3, x^2, x and x^0 .

$$1 = A \quad \text{--- (i)}$$

$$0 = A + B \quad \text{--- (ii)}$$

$$2 = A + B + C \quad \text{--- (iii)}$$

$$2 = B + D \quad \text{--- (iv)}$$

from eq: (i) $A=1$ putting in (ii)

$$0 = 1 + B \Rightarrow B = -1$$

putting value of A and B in (iii)

$$2 = 1 - 1 + C \Rightarrow C = 2$$

putting value of B in eq. (iv)

$$2 = -1 + D \Rightarrow 2 + 1 = D \Rightarrow D = 3$$

hence

$$\frac{x^3+2x+2}{(x^2+x+1)^2} = \frac{(1)x-1}{x^2+x+1} + \frac{2x+3}{(x^2+x+1)^2}$$

$$= \frac{x-1}{x^2+x+1} + \frac{2x+3}{(x^2+x+1)^2}$$

Answer

Q.No2 $\frac{x^2}{(x^2+1)^2(x-1)}$

Consider $\frac{x^2}{(x^2+1)^2(x-1)} = \frac{Ax+B}{x^2+1} + \frac{Cx+D}{(x^2+1)^2} + \frac{E}{x-1}$

Multiplying both sides by $(x^2+1)^2(x-1)$

$$x^2 = (Ax+B)(x^2+1)(x-1) + (Cx+D)(x-1) + E(x^2+1)^2 \quad \text{--- (i)}$$

put $x-1=0 \Rightarrow x=1$ in eq. (i)

$$(1)^2 = 0 + 0 + E((1)^2+1)^2 \Rightarrow 1 = E(2)^2 \Rightarrow 1 = 4E$$

$$\Rightarrow E = \frac{1}{4}$$

equation (i) can be written as

$$x^2 = (Ax+B)(x^3+x-x^2-1) + (Cx+D)(x-1) + E(x^2+2x^2+1)$$

$$\Rightarrow x^2 = A(x^4+x^3-x^2-x) + B(x^3+x-x^2-1) + C(x^2-x) + D(x-1) + E(x^4+2x^2+1)$$

Comparing co-efficients of x^4 , x^3 , x^2 , x and x^0

$$0 = A + E \quad \text{--- (ii)}$$

$$0 = -A + B \quad \text{--- (iii)}$$

$$1 = A - B + C + 2E \quad \text{--- (iv)}$$

$$0 = -A + B - C + D \quad \text{--- (v)}$$

$$0 = -B - D + E \quad \text{--- (vi)}$$

putting $E = \frac{1}{4}$ in eq (ii)

$$0 = A + \frac{1}{4} \Rightarrow \boxed{A = -\frac{1}{4}}$$

putting value of A in (iii)

$$0 = -(-\frac{1}{4}) + B \Rightarrow 0 = \frac{1}{4} + B \Rightarrow \boxed{B = -\frac{1}{4}}$$

putting value of A, B and E in (iv)

$$1 = -\frac{1}{4} - (-\frac{1}{4}) + C + 2(\frac{1}{4})$$

$$\Rightarrow 1 = -\frac{1}{4} + \frac{1}{4} + C + \frac{1}{2} \Rightarrow 1 + \frac{1}{4} - \frac{1}{4} - \frac{1}{2} = C$$

$$\Rightarrow \boxed{C = \frac{1}{2}}$$

putting value of B and E in (vi)

$$0 = -(-\frac{1}{4}) - D + \frac{1}{4} \Rightarrow 0 = \frac{1}{4} - D + \frac{1}{4}$$

$$\Rightarrow 0 = \frac{1}{2} - D \Rightarrow \boxed{D = \frac{1}{2}}$$

$$\begin{aligned} \text{Hence } \frac{x^2}{(x^2+1)^2(x-1)} &= \frac{-\frac{1}{4}x - \frac{1}{4}}{x^2+1} + \frac{\frac{1}{2}x + \frac{1}{2}}{(x^2+1)^2} + \frac{\frac{1}{4}}{x-1} \\ &= \frac{-x-4}{4(x^2+1)} + \frac{x+2}{2(x^2+1)^2} + \frac{1}{4(x-1)} \\ &= -\frac{x+4}{4(x^2+1)} + \frac{x+2}{2(x^2+1)^2} + \frac{1}{4(x-1)} \end{aligned}$$

Answer

Q_{No.3} $\frac{2x-5}{(x^2+2)^2(x-2)}$

Consider $\frac{2x-5}{(x^2+2)^2(x-2)} = \frac{Ax+B}{x^2+2} + \frac{Cx+D}{(x^2+2)^2} + \frac{E}{x-2}$

[Do yourself same as Q_{No.2} you will get
 $A = \frac{1}{36}, B = \frac{1}{18}, C = \frac{1}{6}, D = \frac{7}{3}, E = -\frac{1}{36}$]

Q_{No.4} $\frac{8x^2}{(x^2+1)^2(1-x^2)}$
 $= \frac{8x^2}{(x^2+1)^2(1-x)(1+x)}$

Resolving it into partial fraction.

$$\frac{8x^2}{(x^2+1)^2(1-x)(1+x)} = \frac{Ax+B}{x^2+1} + \frac{Cx+D}{(x^2+1)^2} + \frac{E}{1-x} + \frac{F}{1+x}$$

$$\Rightarrow 8x^2 = (Ax+B)(x^2+1)(1-x)(1+x) + (Cx+D)(1-x)(1+x) + E(x^2+1)^2(1+x) + F(x^2+1)^2(1-x) \quad (i)$$

put $1-x=0 \Rightarrow x=1$ in eq. (i)

$$8(1)^2 = 0 + 0 + E((1)^2+1)^2(1+1) + 0$$

$$\Rightarrow 8 = E(2)^2(2) \Rightarrow 8 = 8E \Rightarrow \boxed{E=1}$$

put $1+x=0 \Rightarrow x=-1$ in eq. (i)

$$8(-1)^2 = 0 + 0 + 0 + F((-1)^2+1)^2(1-(-1))$$

$$\Rightarrow 8(1) = F(1+1)^2(1+1) \Rightarrow 8 = F(4)(2) \Rightarrow \boxed{F=1}$$

Eq. (i) can be written as

$$8x^2 = (Ax+B)(x^2+1)(1-x^2) + (Cx+D)(1-x^2) + E(x^4+2x^2+1)(1+x) + F(x^4+2x^2+1)(1-x)$$

$$\Rightarrow 8x^2 = (Ax+B)(x^2-x^4+1-x^2) + (Cx+D)(1-x^2) + E(x^4+2x^2+1+x^5+2x^3+x) + F(x^4+2x^2+1-x^5-2x^3-x)$$

$$\Rightarrow 8x^2 = (Ax+B)(-x^4+1) + (Cx+D)(1-x^2) + E(x^5+x^4+2x^3+2x^2+1) + F(-x^5+x^4-2x^3+2x^2-x+1)$$

$$\Rightarrow 8x^2 = A(-x^5+x) + B(-x^4+1) + C(x-x^3) + D(1-x^2) + E(x^5+x^4+2x^3+2x^2+1) + F(-x^5+x^4-2x^3+2x^2-x+1)$$

Comparing co-efficients of x^5, x^4, x^3, x^2, x and x^0

$$0 = -A + E - F \quad \text{--- (ii)}$$

$$0 = -B + E + F \quad \text{--- (iii)}$$

$$0 = -C + 2E - 2F \quad \text{--- (iv)}$$

$$8 = -D + 2E + 2F \quad \text{--- (v)}$$

$$0 = A + C + E - F \quad \text{--- (vi)}$$

$$0 = B + D + E + F \quad \text{--- (vii)}$$

putting value of E and F in (ii)

$$0 = -A + 1 - 1 \Rightarrow 0 = -A \Rightarrow \boxed{A = 0}$$

putting value of E and F in (iv)

$$0 = -C + 2(1) - 2(1) \Rightarrow 0 = -C \Rightarrow \boxed{C = 0}$$

putting value of E and F in (iii)

$$0 = -B + 1 + 1 \Rightarrow 0 = -B + 2 \Rightarrow \boxed{B = 2}$$

putting value of E and F in (v)

$$8 = -D + 2(1) + 2(1) \Rightarrow 8 = -D + 4$$

$$\Rightarrow 8 - 4 = -D \Rightarrow 4 = -D \Rightarrow \boxed{D = -4}$$

putting value of

$$\begin{aligned} \text{hence } \frac{8x^2}{(x^2+1)^2(1-x)(1+x)} &= \frac{0 \cdot x + 2}{x^2+1} + \frac{0 \cdot x - 4}{(x^2+1)^2} + \frac{1}{1-x} + \frac{1}{1+x} \\ &= \frac{0+2}{x^2+1} + \frac{0-4}{(x^2+1)^2} + \frac{1}{1-x} + \frac{1}{1+x} \\ &= \frac{2}{x^2+1} - \frac{4}{(x^2+1)^2} + \frac{1}{1-x} + \frac{1}{1+x} \end{aligned}$$

Answer

Q. No. 5

$$\frac{4x^4 + 3x^3 + 6x^2 + 5x}{(x-1)(x^2+x+1)^2}$$

Consider

$$\frac{4x^4 + 3x^3 + 6x^2 + 5x}{(x-1)(x^2+x+1)^2} = \frac{A}{x-1} + \frac{Bx+C}{x^2+x+1} + \frac{Dx+E}{(x^2+x+1)^2}$$

Multiplying both sides by $(x-1)(x^2+x+1)^2$

$$4x^4 + 3x^3 + 6x^2 + 5x = A(x^2+x+1)^2 + (Bx+C)(x-1)(x^2+x+1) + (Dx+E)(x-1) \quad \text{--- (i)}$$

Put $x-1=0 \Rightarrow x=1$ in eq. (i)

$$4(1)^4 + 3(1)^3 + 6(1)^2 + 5(1) = A((1)^2 + 1 + 1)^2 + 0 + 0$$

$$\Rightarrow 4 + 3 + 6 + 5 = A(1 + 1 + 1)^2$$

$$\Rightarrow 18 = 9A \Rightarrow \boxed{A = 2}$$

eq. (i) can be written as

$$4x^4 + 3x^3 + 6x^2 + 5x = A(x^4 + x^2 + 1 + 2x^3 + 2x + 2x^2)$$

$$+ (Bx + C)(x^3 + x^2 + x - x^2 - x - 1) + (Dx + E)(x - 1)$$

$$\Rightarrow 4x^4 + 3x^3 + 6x^2 + 5x = A(x^4 + 2x^3 + 3x^2 + 2x + 1) + (Bx + C)(x^2 - 1) + (Dx + E)(x - 1)$$

$$\Rightarrow 4x^4 + 3x^3 + 6x^2 + 5x = A(x^4 + 2x^3 + 3x^2 + 2x + 1) + B(x^2 - x) + C(x^2 - 1) + D(x^2 - x) + E(x - 1)$$

Comparing coefficients of x^4 , x^3 , x^2 , x and x^0

$$4 = A + B \quad \text{--- (i)}$$

$$3 = 2A + C \quad \text{--- (ii)}$$

$$6 = 3A + D \quad \text{--- (iii)}$$

$$5 = 2A - B - D + E \quad \text{--- (iv)}$$

$$0 = A - C - E \quad \text{--- (v)}$$

put $A = 2$ in eq (i)

$$4 = 2 + B \Rightarrow 4 - 2 = B \Rightarrow \boxed{B = 2}$$

put $A = 2$ in eq (ii)

$$3 = 2(2) + C \Rightarrow 3 - 4 = C \Rightarrow \boxed{C = -1}$$

Put $A = 2$ in eq (iii)

$$6 = 3(2) + D \Rightarrow 6 - 6 = D \Rightarrow \boxed{D = 0}$$

Put $A = 2$ and $C = -1$ in eq (v)

$$0 = 2 - (-1) - E \Rightarrow 0 = 2 + 1 - E \Rightarrow \boxed{E = 3}$$

So

$$\frac{4x^4 + 3x^3 + 6x^2 + 5x}{(x-1)(x^2+x+1)^2} = \frac{2}{x-1} + \frac{2x-1}{x^2+x+1} + \frac{0 \cdot x + 3}{(x^2+x+1)^2}$$

$$= \frac{2}{x-1} + \frac{2x-1}{x^2+x+1} + \frac{3}{(x^2+x+1)^2} \quad \text{Answer}$$

Question # 6:

Do yourself.

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