

$$x^2 + [-(2x+4)]^2 + 4x = 1$$

$$x^2 + (2x+4)^2 + 4x - 1 = 0$$

$$x^2 + 4x^2 + 16x + 16 + 4x - 1 = 0$$

$$5x^2 + 20x + 15 = 0$$

Dividing by 5 we get.

$$x^2 + 4x + 3 = 0$$

$$x^2 + x + 3x + 3 = 0$$

$$x(x+1) + 3(x+1) = 0$$

$$(x+1)(x+3) = 0$$

$$x+1=0, x+3=0 \rightarrow x=-1, x=-3$$

If  $x=-1$  then from eq ③ | If  $x=-3$  then from ③

$$y = -(2(-1)+4)$$

$$y = -(2(-3)+4)$$

$$y = -(-2+4)$$

$$y = -(-6+4)$$

$$y = -2$$

$$y = 2$$

Hence  $\{(-1, -2), (-3, 2)\}$

### EXERCISE 4.8

**Q.1**  $2x - y = 4 \rightarrow$  ①

$$2x^2 - 4xy - y^2 = 6 \rightarrow$$
 ②

From ①  $y = 2x - 4 \rightarrow$  ③

Putting value of  $y$  in equation ②

$$2x^2 - 4x(2x-4) - (2x-4)^2 = 6$$

$$2x^2 - 8x^2 + 16x - (4x^2 - 16x + 16) - 6 = 0$$

$$-6x^2 + 16x - 4x^2 + 16x - 16 - 6 = 0$$

$$-10x^2 + 32x - 22 = 0$$

Dividing by  $-2$  we get

$$5x^2 - 16x - 11 = 0$$

$$5x^2 - 5x - 11x + 11 = 0$$

$$5x(x-1) - 11(x-1) = 0$$

$$(x-1)(5x-11) = 0$$

$$x-1=0, 5x-11=0 \rightarrow x=1, x=11/5$$

If  $x=1$  then from ③ | If  $x=11/5$  then from ③

$$y = 2(1) - 4$$

$$y = 2(11/5) - 4$$

$$y = 2 - 4 = -2$$

$$y = 22/5 - 4$$

$$y = \frac{22-20}{5} = 2/5$$

$$\{(1, -2), (11/5, 2/5)\}$$

**Q.2**  $x + y = 5 \rightarrow$  ①

$$x^2 + 2y^2 = 17 \rightarrow$$
 ②

From ①  $y = 5 - x \rightarrow$  ③

Putting value of  $y$  in equation ②

$$x^2 + 2(5-x)^2 = 17$$

$$x^2 + 2(25 - 10x + x^2) - 17 = 0$$

$$x^2 + 50 - 20x + 2x^2 - 17 = 0$$

$$3x^2 - 20x + 33 = 0$$

$$3x^2 - 9x - 11x + 33 = 0$$

$$3x(x-3) - 11(x-3) = 0$$

$$(x-3)(3x-11) = 0$$

$$x-3=0, 3x-11=0$$

$$x=3, x=11/3$$

If  $x=3$  then from ③ | If  $x=11/3$  then from ③

$$y = 5 - 3$$

$$y = 5 - 11/3$$

$$y = 2$$

$$y = \frac{15-11}{3}$$

$$y = 4/3$$

$$\{(3, 2), (11/3, 4/3)\}$$

**Q.3**  $3x + 2y = 7 \rightarrow$  ①

$$3x^2 = 25 + 2y^2$$

or  $3x^2 - 2y^2 = 25 \rightarrow$  ②

From ①  $2y = 7 - 3x \rightarrow y = \frac{7-3x}{2} \rightarrow$  ③

Putting value of  $y$  in eq. ②

$$3x^2 - 2\left(\frac{7-3x}{2}\right)^2 = 25$$

$$3x^2 - 2\left(\frac{49 - 42x + 9x^2}{4}\right) - 25 = 0$$

$$3x^2 - \left(\frac{9x^2 - 42x + 49}{2}\right) - 25 = 0$$

$$6x^2 - (9x^2 - 42x + 49) - 50 = 0$$

$$6x^2 - 9x^2 + 42x - 49 - 50 = 0$$

$$-3x^2 + 42x - 99 = 0$$

Dividing by -3 we get

$$x^2 - 14x + 33 = 0$$

$$x^2 - 3x - 11x + 33 = 0$$

$$x(x-3) - 11(x-3) = 0$$

$$(x-3)(x-11) = 0$$

$$x-3=0, x-11=0 \Rightarrow x=3, x=11$$

If  $x=3$  then from ③

$$y = \frac{7-3(3)}{2}$$

$$y = \frac{7-9}{2} = \frac{-2}{2} = -1$$

If  $x=11$  then from ③

$$y = \frac{7-3(11)}{2}$$

$$y = \frac{7-33}{2} = \frac{-26}{2}$$

$$y = -13$$

$$\{(3, -1), (11, -13)\}$$

**Q.4**  $x+y=5 \rightarrow ①$

$$\frac{2}{x} + \frac{3}{y} = 2$$

Multiplying by  $xy$  we get

$$\text{or } 2y+3x=2xy \rightarrow ②$$

$$\text{From } ① \ y=5-x \rightarrow ③$$

Putting value of  $y$  in equation ②

$$2(5-x)+3x=2x(5-x)$$

$$10-2x+3x=10x-2x^2$$

$$10+x=10x-2x^2$$

$$2x^2-10x+x+10=0$$

$$2x^2-9x+10=0$$

$$2x^2-4x-5x+10=0$$

$$2x(x-2)-5(x-2)=0$$

$$(x-2)(2x-5)=0$$

$$x-2=0, 2x-5=0$$

$$x=2, x=5/2$$

If  $x=2$  then from ③

$$y=5-2$$

$$y=3$$

If  $x=5/2$  then from ③

$$y=5-\frac{5}{2}$$

$$y=\frac{10-5}{2}$$

$$y=5/2$$

$$\{(2, 3), (5/2, 5/2)\}$$

**Q.5**  $x+y=a+b \rightarrow ①$

$$\frac{a}{x} + \frac{b}{y} = 2$$

$$\text{or } ay+bx=2xy \rightarrow ②$$

$$\text{From } ① \ y=a+b-x \rightarrow ③$$

Putting value of  $y$  in eq. ②

$$a(a+b-x)+bx=2x(a+b-x)$$

$$a^2+ab-ax+bx=2ax+2bx-2x^2$$

$$2x^2-2ax-2bx-ax+bx+a^2+ab=0$$

$$2x^2-3ax-bx+a^2+ab=0$$

$$\text{Using } x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$$x = \frac{-[-(3a+b)] \pm \sqrt{[-(3a+b)]^2 - 4(2)(a^2+ab)}}{2(2)}$$

$$x = \frac{(3a+b) \pm \sqrt{(3a+b)^2 - 8(a^2+ab)}}{4}$$

$$x = \frac{(3a+b) \pm \sqrt{9a^2+b^2+6ab-8a^2-8ab}}{4}$$

$$x = \frac{(3a+b) \pm \sqrt{a^2+b^2-2ab}}{4}$$

$$x = \frac{(3a+b) \pm \sqrt{(a-b)^2}}{4}$$

$$x = \frac{(3a+b) \pm (a-b)}{4}$$

$$x = \frac{3a+b+a-b}{4}, x = \frac{3a+b-(a-b)}{4}$$

$$x = \frac{4a}{4}, x = \frac{3a+b-a+b}{4}$$

$$x=a, x = \frac{2a+2b}{4} \Rightarrow x = \frac{a+b}{2}$$

If  $x=a$  then from ③

$$y=a+b-a$$

$$y=b$$

If  $x = \frac{a+b}{2}$  then from ③

$$y=a+b-\frac{a+b}{2}$$

$$y = \frac{2a+2b-a-b}{2}$$

$$y = \frac{a+b}{2}$$

$$\left\{ (a, b), \left( \frac{a+b}{2}, \frac{a+b}{2} \right) \right\}$$

**Q.6**  $3x + 4y = 25 \rightarrow \textcircled{1}$

$$\frac{3}{x} + \frac{4}{y} = 2$$

or  $3y + 4x = 2xy \rightarrow \textcircled{2}$

From  $\textcircled{1}$   $4y = 25 - 3x \rightarrow y = \frac{25 - 3x}{4} \rightarrow \textcircled{3}$

Putting value of  $y$  in equation  $\textcircled{2}$

$$3 \left( \frac{25 - 3x}{4} \right) + 4x = 2x \left( \frac{25 - 3x}{4} \right)$$

Multiplying by 4 we get

$$3(25 - 3x) + 4x = 2x(25 - 3x)$$

$$75 - 9x + 4x = 50x - 6x^2$$

$$75 + 7x = 50x - 6x^2$$

$$\textcircled{1} \quad 6x^2 - 50x + 7x + 75 = 0$$

$$\textcircled{2} \quad 6x^2 - 43x + 75 = 0$$

$$\textcircled{3} \quad 6x^2 - 18x - 25x + 75 = 0$$

$$6x(x - 3) - 25(x - 3) = 0$$

$$(x - 3)(6x - 25) = 0$$

$$x - 3 = 0, \quad 6x - 25 = 0$$

$$x = 3, \quad x = \frac{25}{6}$$

If  $x = 3$  then from  $\textcircled{3}$  | If  $x = \frac{25}{6}$  then from  $\textcircled{3}$

$$y = \frac{25 - 3(3)}{4}$$

$$y = \frac{25 - 3(\frac{25}{6})}{4}$$

$$\textcircled{1} \quad y = \frac{25 - 9}{4}$$

$$y = \frac{1}{4} \left( 25 - \frac{25}{2} \right)$$

$$y = \frac{16}{4} = 4$$

$$y = \frac{1}{4} \left( \frac{50 - 25}{2} \right)$$

$$y = \frac{1}{4} \cdot \frac{25}{2} = \frac{25}{8}$$

$$\left\{ (3, 4), \left( \frac{25}{6}, \frac{25}{8} \right) \right\}$$

**Q.7**

$$(x - 3)^2 + y^2 = 5$$

$$x^2 - 6x + 9 + y^2 = 5$$

$$\rightarrow x^2 + y^2 - 6x + 4 = 0 \rightarrow \textcircled{1}$$

$$2x = y + 6 \rightarrow \textcircled{2}$$

From  $\textcircled{2}$   $y = 2x - 6 \rightarrow \textcircled{3}$

Putting value of  $y$  in equation  $\textcircled{1}$

$$x^2 + (2x - 6)^2 - 6x + 4 = 0$$

$$x^2 + 4x^2 - 24x + 36 - 6x + 4 = 0$$

$$5x^2 - 30x + 40 = 0$$

Dividing by 5 we get

$$x^2 - 6x + 8 = 0$$

$$x^2 - 2x - 4x + 8 = 0$$

$$x(x - 2) - 4(x - 2) = 0$$

$$(x - 2)(x - 4) = 0$$

$$x - 2 = 0, \quad x - 4 = 0$$

$$x = 2, \quad x = 4$$

If  $x = 2$  then from  $\textcircled{3}$  | If  $x = 4$  then from  $\textcircled{3}$

$$y = 2(2) - 6 = -2, \quad y = 2(4) - 6 = 2$$

$$y = 4 - 6 = -2, \quad y = 8 - 6 = 2$$

$$\left\{ (2, -2), (4, 2) \right\}$$

**Q.8**

$$(x + 3)^2 + (y + 1)^2 = 5$$

or  $x^2 + 6x + 9 + y^2 + 2y + 1 = 5$

$$\textcircled{1} \quad x^2 + y^2 + 6x + 2y + 5 = 0 \rightarrow \textcircled{1}$$

$$x^2 + y^2 + 2x = 4$$

or  $x^2 + y^2 + 2x - 4 = 0$

Subtracting eq.  $\textcircled{2}$  from eq.  $\textcircled{1}$

$$\textcircled{2} \quad x^2 + y^2 + 6x + 2y + 5 = 0$$

$$-x^2 - y^2 + 2x - 4 = 0$$

$$4x - 2y + 14 = 0$$

Dividing by 2 we get

$$2x - y + 7 = 0$$

$$\rightarrow y = 2x + 7 \rightarrow \textcircled{3}$$

Putting value of  $y$  in equation  $\textcircled{2}$

$$x^2 + (2x + 7)^2 + 2x - 4 = 0$$

$$x^2 + 4x^2 + 28x + 49 + 2x - 9 = 0$$

$$5x^2 + 30x + 40 = 0$$

Dividing by 5 we get

$$x^2 + 6x + 8 = 0$$

$$x^2 + 2x + 4x + 8 = 0$$

$$x(x+2) + 4(x+2) = 0$$

$$(x+2)(x+4) = 0$$

$$x+2=0, \quad x+4=0$$

$$x=-2, \quad x=-4$$

If  $x=-2$  then from ③

$$y = 2(-2) + 7$$

$$y = -4 + 7$$

$$y = 3$$

If  $x=-4$  then from ③

$$y = 2(-4) + 7$$

$$y = -8 + 7$$

$$y = -1$$

$$\{(-2, 3), (-4, -1)\}$$

**Q.9**  $x^2 + (y+1)^2 = 18$

$$x^2 + y^2 + 2y + 1 - 18 = 0$$

$$x^2 + y^2 + 2y - 17 = 0 \longrightarrow \textcircled{1}$$

$$(x+2)^2 + y^2 = 21$$

$$x^2 + 4x + 4 + y^2 - 21 = 0$$

$$x^2 + y^2 + 4x - 17 = 0 \longrightarrow \textcircled{2}$$

Subtracting eq ② from eq ①

$$x^2 + y^2 + 2y - 17 = 0$$

$$x^2 + y^2 + 4x - 17 = 0$$

$$\underline{2y - 4x = 0}$$

$$2y - 4x = 0 \Rightarrow 2x = 4x \Rightarrow y = 2x$$

Putting value of  $y$  in equation ②

$$x^2 + (2x)^2 + 4x - 17 = 0$$

$$x^2 + 4x^2 + 4x - 17 = 0$$

$$5x^2 + 4x - 17 = 0$$

Using  $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$x = \frac{-4 \pm \sqrt{(4)^2 - 4(5)(-17)}}{2(5)}$$

$$x = \frac{-4 \pm \sqrt{16 + 340}}{10}$$

$$x = \frac{-4 \pm \sqrt{356}}{10} \Rightarrow x = \frac{-4 \pm \sqrt{4 \times 89}}{10}$$

$$x = \frac{-4 \pm 2\sqrt{89}}{10} \Rightarrow x = \frac{-2 \pm \sqrt{89}}{5}$$

$$x = \frac{-2 \pm \sqrt{89}}{5}$$

If  $x = \frac{-2 + \sqrt{89}}{5}$  then from ③

$$y = 2\left(\frac{-2 + \sqrt{89}}{5}\right)$$

$$y = \frac{-4 + 2\sqrt{89}}{5}$$

$$\left\{\left(\frac{-2 + \sqrt{89}}{5}, \frac{-4 + 2\sqrt{89}}{5}\right), \left(\frac{-2 - \sqrt{89}}{5}, \frac{-4 - 2\sqrt{89}}{5}\right)\right\}$$

If  $x = \frac{-2 - \sqrt{89}}{5}$

Then from ③

$$y = 2\left(\frac{-2 - \sqrt{89}}{5}\right)$$

$$y = \frac{-4 - 2\sqrt{89}}{5}$$

**Q.10**  $x^2 + y^2 + 6x = 1 \longrightarrow \textcircled{1}$

$$x^2 + y^2 + 2(x+y) = 3$$

or  $x^2 + y^2 + 2x + 2y = 3 \longrightarrow \textcircled{2}$

Subtracting equation ② from ①

$$x^2 + y^2 + 6x = 1$$

$$\underline{-x^2 + y^2 + 2x + 2y = 3}$$

$$4x - 2y = -2$$

Dividing by 2 we get

$$2x - y = -1$$

$$\Rightarrow y = 2x + 1$$

Putting values of  $y$  in equation ①

$$x^2 + (2x+1)^2 + 6x = 1$$

$$x^2 + 4x^2 + 4x + 1 + 6x - 1 = 0$$

$$5x^2 + 10x = 0$$

$$5x(x+2) = 0$$

$$5x = 0, \quad x+2 = 0$$

$$x = 0, \quad x = -2$$

If  $x=0$  then from ③

$$y = 2(0) + 1$$

$$y = 1$$

If  $x=-2$  then from ③

$$y = 2(-2) + 1$$

$$y = -4 + 1 = -3$$

$$\{(0, 1), (-2, -3)\}$$