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**Radical** Another word used for root, The sign  $\sqrt{\quad}$  is called radical sign. A number placed to the left of the sign shows the type of root eg.  $\sqrt[2]{\quad}$ ,  $\sqrt[3]{\quad}$ ,  $\sqrt[4]{\quad}$ , ..... ,  $\sqrt[n]{\quad}$  denote square root, Cube roots, Fourth root, ..... ,  $n$ th roots respectively. Here numbers 2, 3, 4 and  $n$  are called index. If there is no number to the left of the sign  $\sqrt{\quad}$ , then root is called square root.

**Radical Equations** The equations involving radical expressions of the variable are called radical equations.

For example;  $\sqrt{2x+8} + \sqrt{x+5} = 7$

$$3x^2 + 2x - \sqrt{3x^2 + 2x - 1} - 7 = 0$$

$\sqrt{x^2 + 2x - 3} + \sqrt{x^2 + 7x - 8} = \sqrt{5x^2 + 15x - 20}$   
are radical equations.

**Extraneous Root** A number obtained in the process of solving an equation, which is actually not a root of the given equation. In other words, we may say that, a root that does not satisfy given equation is called an extraneous root.

\* There are some types of Radical Equations

**Type No. 1**

i) The equation of the form

$$l(ax^2 + bx) + m\sqrt{ax^2 + bx + c} = 0$$

Questions of this type are

Example 1, Q. No. 1, 2, 10

**Example.1**

$$3x^2 + 15x - 2\sqrt{x^2 + 5x + 1} = 2$$

$$3(x^2 + 5x) - 2\sqrt{x^2 + 5x + 1} = 2$$

put  $\sqrt{x^2 + 5x + 1} = y$

$$\rightarrow x^2 + 5x + 1 = y^2 - 1$$

Then given equation becomes

$$3(y^2 - 1) - 2y = 2$$

$$3y^2 - 3 - 2y - 2 = 0$$

$$3y^2 - 2y - 5 = 0$$

$$3y^2 + 3y - 5y - 5 = 0$$

$$3y(y+1) - 5(y+1) = 0$$

$$(y+1)(3y-5) = 0$$

$$y+1 = 0, \quad 3y-5 = 0$$

$$y = -1, \quad y = 5/3$$

If  $y = -1$  then, If  $y = 5/3$  then

$$\sqrt{x^2 + 5x + 1} = -1, \quad \sqrt{x^2 + 5x + 1} = 5/3$$

Squaring both sides

$$x^2 + 5x + 1 = 1, \quad x^2 + 5x + 1 = \frac{25}{9}$$

$$x^2 + 5x + 1 - 1 = 0, \quad 9x^2 + 45x + 9 = 25$$

$$x^2 + 5x = 0, \quad 9x^2 + 45x + 9 - 25 = 0$$

$$x(x+5) = 0, \quad 9x^2 + 45x - 16 = 0$$

$$x = 0, \quad x + 5 = 0, \quad 9x^2 - 3x + 48x - 16 = 0$$

$$x = -5$$

$$3x(3x-1) + 16(3x-1) = 0$$

$$(3x-1)(3x+16) = 0$$

$$3x-1 = 0, \quad 3x+16 = 0$$

$$x = 1/3, \quad x = -16/3$$

On checking we found that 0 and -5 not satisfy given equation, so 0 and -5 are extraneous roots, which can not be written in solution set.

So solution set is  $\{1/3, -16/3\}$

**Type No.2**

When given equation is of the form

$$\sqrt{x+a} + \sqrt{x+b} = \sqrt{x+c}$$

such as. Example 2, Q. No. 3, 4, 5

**Example.2**  $\sqrt{x+8} + \sqrt{x+3} = \sqrt{12x+13}$

Squaring both sides

$$(\sqrt{x+8} + \sqrt{x+3})^2 = (\sqrt{12x+13})^2$$

$$(\sqrt{x+8})^2 + (\sqrt{x+3})^2 + 2\sqrt{x+8}\sqrt{x+3} = 12x+13$$

$$x+8 + x+3 + 2\sqrt{(x+8)(x+3)} = 12x+13$$

$$2x+11 + 2\sqrt{x^2+3x+8x+24} = 12x+13$$

$$2\sqrt{x^2+11x+24} = 12x-2x+13-11$$

$$2\sqrt{x^2+11x+24} = 10x+2$$

$$2\sqrt{x^2+11x+24} = 2(5x+1)$$

$$\sqrt{x^2+11x+24} = 5x+1$$

Squaring again

$$(\sqrt{x^2+11x+24})^2 = (5x+1)^2$$

$$x^2+11x+24 = 25x^2+10x+1$$

$$25x^2-x^2+10x-11x+1-24 = 0$$

$$24x^2-x-23 = 0$$

$$24x^2-24x+23x-23 = 0$$

$$24x(x-1) + 23(x-1) = 0$$

$$(x-1)(24x+23) = 0$$

$$x-1 = 0, \quad 24x+23 = 0$$

$$x = 1, \quad x = -23/24$$

On checking we found that  $-23/24$  is an extraneous root.

Hence solution set is  $\{1\}$ .

OR. S.S.  $\{1\}$ .

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**Type No .3**

When given equation is of the form

$$\sqrt{ax^2+bx+c} + \sqrt{px^2+qx+r} = \sqrt{lx^2+mx+n}$$

such as. Example 3, Q. No. 7, 8, 9

**Example.3**

$$\sqrt{x^2+4x-21} + \sqrt{x^2-x-6} = \sqrt{6x^2-5x-39}$$

Factorizing the expression under radical sign.

$$\sqrt{x^2+7x-3x-21} + \sqrt{x^2+2x-3x-6} = \sqrt{6x^2-18x+13x-39}$$

$$\sqrt{x(x+7)-3(x-7)} + \sqrt{x(x+2)-3(x+2)} = \sqrt{6x(x-3)+13(x-3)}$$

$$\sqrt{(x-3)(x+7)} + \sqrt{(x-3)(x+2)} = \sqrt{(x-3)(6x+13)}$$

$$\sqrt{(x-3)(x+7)} + \sqrt{(x-3)(x+2)} - \sqrt{(x-3)(6x+13)} = 0$$

$$\sqrt{x-3} \{ \sqrt{x+7} + \sqrt{x+2} - \sqrt{6x+13} \} = 0$$

$$\sqrt{x-3} = 0, \quad \sqrt{x+7} + \sqrt{x+2} - \sqrt{6x+13} = 0$$

$$x-3=0, \quad \sqrt{x+7} + \sqrt{x+2} = \sqrt{6x+13}$$

$$x=3$$

Squaring on both sides

$$(\sqrt{x+7} + \sqrt{x+2})^2 = (\sqrt{6x+13})^2$$

$$(\sqrt{x+7})^2 + (\sqrt{x+2})^2 + 2\sqrt{x+7}\sqrt{x+2} = 6x+13$$

$$x+7 + x+2 + 2\sqrt{(x+7)(x+2)} = 6x+13$$

$$2x+9 + 2\sqrt{x^2+2x+7x+14} = 6x+13$$

$$2\sqrt{x^2+9x+14} = 6x-2x+13-9$$

$$2\sqrt{x^2+9x+14} = 4x+4$$

$$\sqrt{x^2+9x+14} = 2x+2$$

Squaring again

$$(\sqrt{x^2+9x+14})^2 = (2x+2)^2$$

$$x^2+9x+14 = 4x^2+8x+4$$

$$4x^2 - x^2 + 8x - 9x + 4 - 4 = 0$$

$$3x^2 - x - 10 = 0$$

$$3x^2 - 6x + 5x - 10 = 0$$

$$3x(x-2) + 5(x-2) = 0$$

$$(x-2)(3x+5) = 0$$

$$x-2=0, \quad 3x+5=0$$

$$x=2, \quad x=-5/3$$

On checking  $-5/3$  is to be found

as an extraneous root.

So S.S is  $\{3, 2\}$ .

**Type No.4**

When given equations are of the form

$$\sqrt{ax^2+bx+c} + \sqrt{px^2+qx+r} = mx+n$$

such as Example.4, Q. 6, 11, 12

**Example.4**

$$\sqrt{3x^2-7x-30} - \sqrt{2x^2-7x-5} = x-5$$

put  $\sqrt{3x^2-7x-30} = a, \quad \sqrt{2x^2-7x-5} = b$

Then given equation becomes

$$a - b = x - 5 \rightarrow \textcircled{1}$$

To find the values of  $a$  and  $b$

we find  $a^2 - b^2$  as;

$$a^2 - b^2 = (\sqrt{3x^2-7x-30})^2 - (\sqrt{2x^2-7x-5})^2$$

$$a^2 - b^2 = 3x^2 - 7x - 30 - (2x^2 - 7x - 5)$$

$$a^2 - b^2 = 3x^2 - 7x - 30 - 2x^2 + 7x + 5$$

$$a^2 - b^2 = x^2 - 25$$

$$a^2 - b^2 = (x)^2 - (5)^2$$

$$\therefore a^2 - b^2 = (a-b)(a+b)$$

$$(a-b)(a+b) = (x-5)(x+5) \rightarrow \textcircled{2}$$

Putting value  $a-b=x-5$  from eq ① in eq ②

$$(x-5)(a+b) = (x-5)(x+5)$$

$$\rightarrow a+b = x+5 \rightarrow \textcircled{3}$$

Adding ① and ③

$$a-b = x-5$$

$$a+b = x+5$$

$$\hline 2a = 2x$$

$$\rightarrow a = x$$

Putting value of  $a$  in equation ①

$$x-b = x-5$$

$$\rightarrow b = 5$$

If  $a = x$  Then

$$\sqrt{3x^2 - 7x - 30} = x$$

Squaring

$$3x^2 - 7x - 30 = x^2$$

$$3x^2 - x^2 - 7x - 30 = 0$$

$$2x^2 - 7x - 30 = 0$$

If  $b = 5$  Then

$$\sqrt{2x^2 - 7x - 5} = 5$$

Squaring

$$2x^2 - 7x - 5 = 25$$

$$2x^2 - 7x - 5 - 25 = 0$$

$$2x^2 - 7x - 30 = 0$$

Solving any one

$$2x^2 - 7x - 30 = 0$$

$$2x^2 - 12x + 5x - 30 = 0$$

$$2x(x-6) + 5(x-6) = 0$$

$$(x-6)(2x+5) = 0$$

$$x-6=0, \quad 2x+5=0$$

$$x = 6, \quad x = -5/3$$

On checking we found  $-5/3$  as an extraneous root

So S.S is  $\{6\}$ .

## EXERCISE 4.3

### Q.1 $3x^2 + 2x - \sqrt{3x^2 + 2x - 1} = 7$

put  $\sqrt{3x^2 + 2x - 1} = y$

$$\rightarrow 3x^2 + 2x - 1 = y^2$$

$$3x^2 + 2x = y^2 + 1$$

Then given equation takes form

$$y^2 + 1 - y = 7$$

$$y^2 - y + 1 - 7 = 0$$

$$y^2 - y - 6 = 0$$

$$y^2 + 2y - 3y - 6 = 0$$

$$y(y+2) - 3(y+2) = 0$$

$$(y+2)(y-3) = 0$$

$$y+2=0, \quad y-3=0$$

$$y = -2, \quad y = 3$$

If  $y = -2$  Then,

$$\sqrt{3x^2 + 2x - 1} = -2$$

$$3x^2 + 2x - 1 = 4$$

$$3x^2 + 2x - 1 - 4 = 0$$

$$3x^2 + 2x - 5 = 0$$

$$3x^2 - 3x + 5x - 5 = 0$$

$$3x(x-1) + 5(x-1) = 0$$

$$(x-1)(3x+5) = 0$$

$$x-1=0, \quad 3x+5=0$$

$$x = 1, \quad x = -5/3$$

If  $y = 3$  Then

$$\sqrt{3x^2 + 2x - 1} = 3$$

$$3x^2 + 2x - 1 = 9$$

$$3x^2 + 2x - 1 - 9 = 0$$

$$3x^2 + 2x - 10 = 0$$

$$\text{Using } x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-2 \pm \sqrt{(2)^2 - 4(3)(-10)}}{2(3)}$$

$$x = \frac{-2 \pm \sqrt{4 + 120}}{6}$$

$$x = \frac{-2 \pm \sqrt{124}}{6}$$

$$x = \frac{-2 \pm \sqrt{4 \times 31}}{6} \Rightarrow x = \frac{-2 \pm 2\sqrt{31}}{6}$$

$$x = 2 \left( \frac{-1 \pm \sqrt{31}}{6} \right) \Rightarrow x = \frac{-1 \pm \sqrt{31}}{3}$$

On checking we found that  $1$  and  $-5/3$  are extraneous roots. Hence

$$\text{S.S is } \left\{ \frac{-1 \pm \sqrt{31}}{3} \right\}$$

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$$\mathbf{Q.2} \quad x^2 - \frac{x}{2} - 7 = x - 3\sqrt{2x^2 - 3x + 2}$$

Multiplying by 2

$$2x^2 - x - 14 = 2x - 6\sqrt{2x^2 - 3x + 2}$$

$$2x^2 - x - 2x - 14 + 6\sqrt{2x^2 - 3x + 2} = 0$$

$$2x^2 - 3x - 14 + 6\sqrt{2x^2 - 3x + 2} = 0$$

put  $\sqrt{2x^2 - 3x + 2} = y$

$$\rightarrow 2x^2 - 3x + 2 = y^2$$

$$2x^2 - 3x = y^2 - 2$$

Given equation takes form

$$y^2 - 2 - 14 + 6y = 0$$

$$y^2 + 6y - 16 = 0$$

$$y^2 - 2y + 8y - 16 = 0$$

$$y(y-2) + 8(y-2) = 0$$

$$(y-2)(y+8) = 0$$

$$y-2=0, \quad y+8=0$$

$$y=2, \quad y=-8$$

If  $y=2$  then

$$\sqrt{2x^2 - 3x + 2} = 2$$

Squaring

$$2x^2 - 3x + 2 = 4$$

$$2x^2 - 3x + 2 - 4 = 0$$

$$2x^2 - 3x - 2 = 0$$

$$2x^2 + x - 4x - 2 = 0$$

$$x(2x+1) - 2(2x+1) = 0$$

$$(x-2)(2x+1) = 0$$

$$x-2=0, \quad 2x+1=0$$

$$x=2, \quad x=-\frac{1}{2}$$

$$x = \frac{3 \pm \sqrt{505}}{4}$$

On checking we found  $\frac{3 \pm \sqrt{505}}{4}$  is an extraneous root.

Hence S.S  $\left\{ 2, -\frac{1}{2} \right\}$

$$\mathbf{Q.3} \quad \sqrt{2x+8} + \sqrt{x+5} = 7$$

Squaring both sides

$$(\sqrt{2x+8} + \sqrt{x+5})^2 = (7)^2$$

$$(\sqrt{2x+8})^2 + (\sqrt{x+5})^2 + 2\sqrt{2x+8}\sqrt{x+5} = 49$$

$$2x+8 + x+5 + 2\sqrt{(2x+8)(x+5)} = 49$$

$$3x+13 + 2\sqrt{2x^2+10x+8x+40} = 49$$

$$2\sqrt{2x^2+18x+40} = 49-13-3x$$

$$2\sqrt{2x^2+18x+40} = 36-3x$$

Squaring both sides again

$$4(2x^2+18x+40) = (36-3x)^2$$

$$8x^2+72x+160 = 1296+9x^2-216x$$

$$9x^2-8x^2-216x-72x+1296-160=0$$

$$x^2-288x+1136=0$$

$$x^2-4x-284x+1136=0$$

$$x(x-4)-284(x-4)=0$$

$$(x-4)(x-284)=0$$

$$x-4=0, \quad x-284=0$$

$$x=4, \quad x=284$$

On checking we found 284 is an extraneous root. So

S.S is  $\{ 4 \}$

$$\mathbf{Q.4} \quad \sqrt{3x+4} = 2 + \sqrt{2x-4}$$

$$\sqrt{3x+4} - \sqrt{2x-4} = 2$$

Squaring both sides

$$(\sqrt{3x+4} - \sqrt{2x-4})^2 = (2)^2$$

$$(\sqrt{3x+4})^2 + (\sqrt{2x-4})^2 - 2\sqrt{3x+4}\sqrt{2x-4} = 4$$

$$3x+4 + 2x-4 - 2\sqrt{(3x+4)(2x-4)} = 4$$

$$5x - 2\sqrt{6x^2-12x+8x-16} = 4$$

$$-2\sqrt{6x^2-4x-16} = 4-5x$$

squaring again

$$4(6x^2-4x-16) = (4-5x)^2$$

$$24x^2-16x-64 = 16+25x^2-40x$$

$$25x^2-24x^2-40x+16x+16+64=0$$

$$x^2-24x+80=0$$

$$x^2-4x-20x+80=0$$

$$x(x-4)-20(x-4)=0$$

$$(x-4)(x-20)=0$$

$$x-4=0, \quad x-20=0$$

$$x=4, \quad x=20$$

On checking we found that no root is an extraneous root.

Hence S.S  $\{4, 20\}$

**Q.5**  $\sqrt{x+7} + \sqrt{x+2} = \sqrt{6x+13}$

squaring on both sides.

$$(\sqrt{x+7} + \sqrt{x+2})^2 = (\sqrt{6x+13})^2$$

$$(\sqrt{x+7})^2 + (\sqrt{x+2})^2 + 2\sqrt{x+7}\sqrt{x+2} = 6x+13$$

$$x+7+x+2+2\sqrt{(x+7)(x+2)} = 6x+13$$

$$2x+9+2\sqrt{x^2+2x+7x+14} = 6x+13$$

$$2\sqrt{x^2+9x+14} = 6x-2x+13-9$$

$$2\sqrt{x^2+9x+14} = 4x+4$$

$$\sqrt{x^2+9x+14} = 2x+2$$

squaring again

$$(\sqrt{x^2+9x+14})^2 = (2x+2)^2$$

$$x^2+9x+14 = 4x^2+8x+4$$

$$4x^2-x^2+8x-9x+4-14=0$$

$$3x^2-x-10=0$$

$$3x^2-6x+5x-10=0$$

$$3x(x-2)+5(x-2)=0$$

$$(x-2)(3x+5)=0$$

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$$x-2=0, \quad 3x+5=0$$

$$x=2, \quad x=-5/3$$

On checking we found that  $-5/3$  is an extraneous root. Hence

S.S  $\{2\}$

**Q6**  $\sqrt{x^2+x+1} - \sqrt{x^2+x-1} = 1$

put  $\sqrt{x^2+x+1} = a, \sqrt{x^2+x-1} = b$

Given equation takes form

$$a-b=1 \rightarrow \textcircled{1}$$

To find a and b we find  $a^2-b^2$  as

$$a^2-b^2 = (\sqrt{x^2+x+1})^2 - (\sqrt{x^2+x-1})^2$$

$$a^2-b^2 = x^2+x+1 - (x^2+x-1)$$

$$a^2-b^2 = x^2+x+1 - x^2-x+1$$

$$a^2-b^2 = 2 \quad \therefore a^2-b^2=(a-b)(a+b)$$

$$(a-b)(a+b)=2 \rightarrow \textcircled{2}$$

putting value of a-b from  $\textcircled{1}$  in  $\textcircled{2}$

$$1(a+b)=2 \rightarrow a+b=2 \rightarrow \textcircled{3}$$

Adding  $\textcircled{1}$  and  $\textcircled{3}$

$$a-b=1$$

$$a+b=2$$

$$2a=3 \rightarrow a=3/2$$

putting values of a in  $\textcircled{1}$

$$\frac{3}{2}-1=b \rightarrow b=\frac{3-2}{2}=\frac{1}{2}$$

if  $a=3/2$  then

$$\sqrt{x^2+x+1} = 3/2$$

$$2\sqrt{x^2+x+1} = 3$$

squaring

$$4(x^2+x+1) = 9$$

$$4x^2+4x+4-9=0$$

$$4x^2+4x-5=0$$

if  $b=1/2$  then

$$\sqrt{x^2+x-1} = 1/2$$

$$2\sqrt{x^2+x-1} = 1$$

squaring

$$4(x^2+x-1) = 1$$

$$4x^2+4x-4-1=0$$

$$4x^2+4x-5=0$$

Solving any one

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$$4x^2 + 4x - 5 = 0$$

Using  $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$x = \frac{-4 \pm \sqrt{(4)^2 - 4(4)(-5)}}{2(4)}$$

$$x = \frac{-4 \pm \sqrt{16+80}}{8} \Rightarrow x = \frac{-4 \pm \sqrt{96}}{8}$$

$$x = \frac{-4 \pm \sqrt{16 \times 6}}{8} \Rightarrow x = \frac{-4 \pm 4\sqrt{6}}{8}$$

$$x = \frac{4(-1 \pm \sqrt{6})}{8} \Rightarrow x = \frac{-1 \pm \sqrt{6}}{2}$$

On checking we found that no root is an extraneous root. So

S.S  $\left\{ \frac{-1 \pm \sqrt{6}}{2} \right\}$

**Q.7**

$$\sqrt{x^2+2x-3} + \sqrt{x^2+7x-8} = \sqrt{5(x^2+3x-4)}$$

Factorizing the expressions under the radical sign.

$$\sqrt{x^2-x+3x-3} + \sqrt{x^2-x+8x-8} = \sqrt{5(x^2-x+4x-4)}$$

$$\sqrt{x(x-1)+3(x-1)} + \sqrt{x(x-1)+8(x-1)} = \sqrt{5[x(x-1)+4(x-1)]}$$

$$\sqrt{(x-1)(x+3)} + \sqrt{(x-1)(x+8)} = \sqrt{5(x-1)(x+4)}$$

$$\sqrt{(x-1)(x+3)} + \sqrt{(x-1)(x+8)} - \sqrt{(x-1)(x+4)} \cdot 5 = 0$$

$$\sqrt{x-1} \left\{ \sqrt{x+3} + \sqrt{x+8} - \sqrt{5(x+4)} \right\} = 0$$

$$\sqrt{x-1} = 0, \sqrt{x+3} + \sqrt{x+8} - \sqrt{5(x+4)} = 0$$

$$x-1=0, \sqrt{x+3} + \sqrt{x+8} = \sqrt{5(x+4)}$$

$$x=1, \text{ Squaring both sides}$$

$$\left( \sqrt{x+3} + \sqrt{x+8} \right)^2 = \left( \sqrt{5(x+4)} \right)^2$$

$$\left( \sqrt{x+3} \right)^2 + \left( \sqrt{x+8} \right)^2 + 2\sqrt{x+3}\sqrt{x+8} = 5(x+4)$$

$$x+3 + x+8 + 2\sqrt{(x+3)(x+8)} = 5x+20$$

$$2x+11 + 2\sqrt{x^2+8x+3x+24} = 5x+20$$

$$2\sqrt{x^2+11x+24} = 5x-2x+20-11$$

$$2\sqrt{x^2+11x+24} = 3x+9$$

Squaring again

$$4(x^2+11x+24) = (3x+9)^2$$

$$4x^2+44x+96 = 9x^2+54x+81$$

$$9x^2-4x^2+54x-44x+81-96=0$$

$$5x^2+10x-15=0$$

Dividing by 5

$$x^2+2x-3=0$$

$$x^2-x+3x-3=0$$

$$x(x-1)+3(x-1)=0$$

$$(x-1)(x+3)=0$$

$$x-1=0, x+3=0$$

$$x=1, x=-3$$

On checking we found that no root is extraneous root. So

S.S is  $\{1, -3\}$ .

**Q.8**

$$\sqrt{2x^2-5x-3} + 3\sqrt{2x+1} = \sqrt{2x^2+25x+12}$$

Factorizing the expressions under radical sign.

$$\sqrt{2x^2+x-6x-3} + 3\sqrt{2x+1} = \sqrt{2x^2+x+24x+12}$$

$$\sqrt{x(2x+1)-3(2x+1)} + 3\sqrt{2x+1} = \sqrt{x(2x+1)+12(2x+1)}$$

$$\sqrt{(2x+1)(x-3)} + 3\sqrt{2x+1} - \sqrt{(2x+1)(x+12)} = 0$$

$$\sqrt{2x+1} \left\{ \sqrt{x-3} + 3 - \sqrt{x+12} \right\} = 0$$

$$\sqrt{2x+1} = 0, \sqrt{x-3} + 3 - \sqrt{x+12} = 0$$

$$2x+1=0, \sqrt{x-3} + 3 = \sqrt{x+12}$$

$$x = -\frac{1}{2}, \text{ Squaring}$$

$$\left( \sqrt{x-3} + 3 \right)^2 = \left( \sqrt{x+12} \right)^2$$

$$\left( \sqrt{x-3} \right)^2 + (3)^2 + 2(3)\sqrt{x-3} = x+12$$

$$x-3+9+6\sqrt{x-3} = x+12$$

$$x+6+6\sqrt{x-3} = x+12$$

$$6\sqrt{x-3} = x-x+12-6$$

$$6\sqrt{x-3} = 6 \Rightarrow \sqrt{x-3} = 1$$

$$x - 3 = 1 \Rightarrow x = 4$$

On checking we found no root is an extraneous root. Hence

$$S.S \text{ is } \left\{ -\frac{1}{2}, 4 \right\}$$

**Q.9**

$\sqrt{3x^2 - 5x + 2} + \sqrt{6x^2 - 11x + 5} = \sqrt{5x^2 - 9x + 4}$   
Factorizing the expressions under radical sign.

$$\sqrt{3x^2 - 3x - 2x + 2} + \sqrt{6x^2 - 6x - 5x + 5} = \sqrt{5x^2 - 5x - 4x + 4}$$

$$\sqrt{3x(x-1) - 2(x-1)} + \sqrt{6x(x-1) - 5(x-1)} = \sqrt{5x(x-1) - 4(x-1)}$$

$$\sqrt{(x-1)(3x-2)} + \sqrt{(x-1)(6x-5)} = \sqrt{(x-1)(5x-4)}$$

$$\sqrt{(x-1)(3x-2)} + \sqrt{(x-1)(6x-5)} - \sqrt{(x-1)(5x-4)} = 0$$

$$\sqrt{x-1} \{ \sqrt{3x-2} + \sqrt{6x-5} - \sqrt{5x-4} \} = 0$$

$$\sqrt{x-1} = 0, \quad \sqrt{3x-2} + \sqrt{6x-5} - \sqrt{5x-4} = 0$$

$$x-1 = 0, \quad \sqrt{3x-2} + \sqrt{6x-5} = \sqrt{5x-4}$$

$$x = 1$$

Squaring both sides

$$(\sqrt{3x-2} + \sqrt{6x-5})^2 = (\sqrt{5x-4})^2$$

$$(\sqrt{3x-2})^2 + (\sqrt{6x-5})^2 + 2\sqrt{3x-2}\sqrt{6x-5} = 5x-4$$

$$3x-2 + 6x-5 + 2\sqrt{(3x-2)(6x-5)} = 5x-4$$

$$9x-7 + 2\sqrt{18x^2 - 15x - 12x + 10} = 5x-4$$

$$2\sqrt{18x^2 - 27x + 10} = 5x - 9x - 4 + 7$$

$$2\sqrt{18x^2 - 27x + 10} = 3 - 4x$$

Squaring again

$$4(18x^2 - 27x + 10) = (3 - 4x)^2$$

$$72x^2 - 108x + 40 = 9 + 16x^2 - 24x$$

$$72x^2 - 16x^2 - 108x + 24x + 40 - 9 = 0$$

$$56x^2 - 84x + 31 = 0$$

Using  $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$x = \frac{-(-84) \pm \sqrt{(-84)^2 - 4(56)(31)}}{2(56)}$$

$$x = \frac{84 \pm \sqrt{7056 - 6944}}{112} \Rightarrow x = \frac{84 \pm \sqrt{112}}{112}$$

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$$x = \frac{84 \pm \sqrt{16 \times 7}}{112}$$

$$x = \frac{84 \pm 4\sqrt{7}}{112} \Rightarrow x = 4 \left( \frac{21 \pm \sqrt{7}}{112} \right)$$

$$x = \frac{21 \pm \sqrt{7}}{28}$$

On checking we found that no root is an extraneous root. So

$$S.S \text{ is } \left\{ \frac{21 \pm \sqrt{7}}{28} \right\}$$

**Q.10**

$$(x+4)(x+1) = \sqrt{x^2 + 2x - 15} + 3x + 31$$

$$x^2 + x + 4x + 4 = \sqrt{x^2 + 2x - 15} + 3x + 31$$

$$x^2 + 5x + 4 = \sqrt{x^2 + 2x - 15} + 3x + 31$$

$$x^2 + 5x - 3x + 4 - 31 - \sqrt{x^2 + 2x - 15} = 0$$

$$x^2 + 2x - 27 - \sqrt{x^2 + 2x - 15} = 0$$

put  $\sqrt{x^2 + 2x - 15} = y$

$$\Rightarrow x^2 + 2x - 15 = y^2$$

$$x^2 + 2x = y^2 + 15$$

Given equation takes form

$$y^2 + 15 - 27 - y = 0$$

$$y^2 - y - 12 = 0$$

$$y^2 + 3y - 4y - 12 = 0$$

$$y(y+3) - 4(y+3) = 0$$

$$(y-4)(y+3) = 0$$

$$y-4 = 0, \quad y+3 = 0$$

$$y = 4, \quad y = -3$$

If  $y = 4$  then

$$\sqrt{x^2 + 2x - 15} = 4$$

$$x^2 + 2x - 15 = 16$$

$$x^2 + 2x - 15 - 16 = 0$$

$$x^2 + 2x - 31 = 0$$

$$a=1, b=2, c=-31$$

Using  $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

If  $y = -3$  then

$$\sqrt{x^2 + 2x - 15} = -3$$

$$x^2 + 2x - 15 = 9$$

$$x^2 + 2x - 15 - 9 = 0$$

$$x^2 + 2x - 24 = 0$$

$$x^2 - 4x + 6x - 24 = 0$$

$$x(x-4) + 6(x-4) = 0$$

$$x = \frac{-2 \pm \sqrt{(2)^2 - 4(1)(-3)}}{2(1)}$$

$$x = \frac{-2 \pm \sqrt{4+124}}{2}$$

$$x = \frac{-2 \pm \sqrt{128}}{2}$$

$$x = \frac{-2 \pm 8\sqrt{2}}{2}$$

$$x = -1 \pm 4\sqrt{2}$$

$$(x-4)(x+6) = 0$$

$$x-4=0, x+6=0$$

$$x=4, x=-6$$

$$\rightarrow x = \frac{-2 \pm \sqrt{64 \times 2}}{2}$$

$$\rightarrow x = \frac{2(-1 \pm 4\sqrt{2})}{2}$$

$$x = -1 \pm 4\sqrt{2}$$

On checking we found that 4 and -6 are extraneous roots. Hence

$$\text{S.S is } \{-1 \pm 4\sqrt{2}\}$$

### Q.11

$$\sqrt{3x^2-2x+9} + \sqrt{3x^2-2x-4} = 13$$

$$\text{put } \sqrt{3x^2-2x+9} = a, \sqrt{3x^2-2x-4} = b$$

Given equation takes form

$$a+b = 13 \rightarrow \textcircled{1}$$

To find a and b we find  $a^2 - b^2$  as

$$a^2 - b^2 = (\sqrt{3x^2-2x+9})^2 - (\sqrt{3x^2-2x-4})^2$$

$$a^2 - b^2 = 3x^2 - 2x + 9 - (3x^2 - 2x - 4)$$

$$a^2 - b^2 = 3x^2 - 2x + 9 - 3x^2 + 2x + 4$$

$$a^2 - b^2 = 13 \quad \therefore a^2 - b^2 = (a+b)(a-b)$$

$$(a+b)(a-b) = 13 \rightarrow \textcircled{2}$$

Putting value of a+b from ① in ②

$$13(a-b) = 13 \rightarrow a-b = 1 \rightarrow \textcircled{3}$$

Adding ① and ③

$$\begin{array}{r} a+b = 13 \\ a-b = 1 \\ \hline \end{array}$$

$$2a = 14 \rightarrow a = 7$$

Putting value of a in ①

$$7+b = 13 \rightarrow b = 6$$

a = 7 then

if b = 6 then

$$\sqrt{3x^2-2x+9} = 7 \quad \sqrt{3x^2-2x-4} = 6$$

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$$3x^2 - 2x + 9 = 49 \quad | \quad 3x^2 - 2x - 4 = 36$$

$$3x^2 - 2x + 9 - 49 = 0 \quad | \quad 3x^2 - 2x - 4 - 36 = 0$$

$$3x^2 - 2x - 40 = 0 \quad | \quad 3x^2 - 2x - 40 = 0$$

Solving any one

$$3x^2 - 2x - 40 = 0$$

$$3x^2 - 12x + 10x - 40 = 0$$

$$3x(x-4) + 10(x-4) = 0$$

$$(x-4)(3x+10) = 0$$

$$x-4=0, \quad 3x+10=0$$

$$x = 4, \quad x = -10/3$$

On checking we found no root is extraneous root. So.

$$\text{S.S is } \{4, -10/3\}$$

### Q.12

$$\sqrt{5x^2+7x+2} - \sqrt{4x^2+7x+18} = x-4$$

$$\text{put } \sqrt{5x^2+7x+2} = a, \sqrt{4x^2+7x+18} = b$$

Given equation takes form

$$a-b = x-4 \rightarrow \textcircled{1}$$

To find a and b we find  $a^2 - b^2$  as

$$a^2 - b^2 = (\sqrt{5x^2+7x+2})^2 - (\sqrt{4x^2+7x+18})^2$$

$$a^2 - b^2 = 5x^2 + 7x + 2 - (4x^2 + 7x + 18)$$

$$a^2 - b^2 = 5x^2 + 7x + 2 - 4x^2 - 7x - 18$$

$$a^2 - b^2 = x^2 - 16$$

$$a^2 - b^2 = (x)^2 - (4)^2 \quad \therefore a^2 - b^2 = (a-b)(a+b)$$

$$(a-b)(a+b) = (x-4)(x+4) \rightarrow \textcircled{2}$$

Putting value of a-b from ① in ②

$$(x-4)(a+b) = (x-4)(x+4)$$

$$\rightarrow a+b = x+4 \rightarrow \textcircled{3}$$

Adding eq ① and ③

$$a-b = x-4$$

$$a+b = x+4$$

$$\begin{array}{r} 2a = 2x \\ \hline \end{array} \rightarrow a = x$$

Putting value of a in ①

$$x-b = x-4 \rightarrow b = 4$$

<p>If <math>a = x</math> Then</p> $\sqrt{5x^2 + 7x + 2} = x$ $5x^2 + 7x + 2 = x^2$ $5x^2 - x^2 + 7x + 2 = 0$ $4x^2 + 7x + 2 = 0$	<p>If <math>b = 4</math> Then</p> $\sqrt{4x^2 + 7x + 18} = 4$ $4x^2 + 7x + 18 = 16$ $4x^2 + 7x + 18 - 16 = 0$ $4x^2 + 7x + 2 = 0$
--	---

Solving any one

$$4x^2 + 7x + 2 = 0$$

Using  $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$x = \frac{-7 \pm \sqrt{(7)^2 - 4(4)(2)}}{2(4)}$$

$$x = \frac{-7 \pm \sqrt{49 - 32}}{8} \Rightarrow x = \frac{-7 \pm \sqrt{17}}{8}$$

On checking we found that no root is extraneous root. Hence (i)

S.S is  $\left\{ \frac{-7 \pm \sqrt{17}}{8} \right\}$

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