$$x = \frac{-(a+2b+c) \pm \sqrt{(a-c)^2}}{2(a+b)}$$

$$x = \frac{-(a+2b+c) \pm (a-c)}{2(a+b)}$$

$$x = \frac{-(a+2b+c)+a-c}{2(a+b)} \quad , \quad x = \frac{-(a+2b+c)-a+c}{2(a+b)}$$

$$x = \frac{-a-2b-c+a-c}{2(a+b)} \quad , \quad x = \frac{-a-2b-c-a+c}{2(a+b)}$$

$$x = \frac{-2b-2c}{2(a+b)} \quad , \quad x = \frac{-2b-2a}{2(a+b)}$$

$$x = \frac{-2(b+c)}{2(a+b)} \quad , \quad x = \frac{-2(a+b)}{2(a+b)}$$

$$x = -\frac{(b+c)}{a+b} \quad , \quad x = -1$$

$$\left\{ -\frac{(b+c)}{a+b} \, , -1 \right\}$$

Equations Reducible to Quadratic Form

Some times we come across equations of type which do not look to be quadratic, but they can be reduced to the quadratic form. There are such five types.

Type No.1

When given equation is of the form

$$ax^{2n} + bx^n + c = 0$$

OR. $$a(x^n)^2 + bx^n + c = 0$$

$\boxed{\text{We put } x^n = y}$

Following questions are of Type. No. 1

Example. No. 1, Q. No. 1, 2, 3, 4, 5

Example 1

$$x^{1/2} - x^{1/4} - 6 = 0$$

We may write as;

$$x^{1/4 \times 2} - x^{1/4} - 6 = 0$$

$$(x^{1/4})^2 - x^{1/4} - 6 = 0 \qquad \because a^{m \times n} = (a^m)^n$$

put $x^{1/4} = y$ then

$$y^2 - y - 6 = 0$$

Factorizing;

$-6y^2$

$2y \quad -3y$

$$y^2 + 2y - 3y - 6 = 0$$

$$y(y+2) - 3(y+2) = 0$$

$$(y+2)(y-3) = 0$$

$$y+2 = 0 \ , \ y - 3 = 0$$

$$y = -2 \ , \ y = 3$$

If $y = -2$, If $y = 3$

Then $x^{1/4} = -2$ Then $x^{1/4} = 3$

$$(x^{1/4})^4 = (-2)^4 \ , \ (x^{1/4})^4 = (3)^4$$

$$x = 16 \ , \ x = 81$$

$$\{16, 81\}$$

Type No.2

When given equation is of the form

$$(x+a)(x+b)(x+c)(x+d) = k$$

where $a + c = b + d$

Then we re-arrange given equation as

$$\{(x+a)(x+c)\} . \{(x+b)(x+d)\} = k$$

In this way first two terms become same we put them y and solve further

Questions of such type are as;

Example. 2, Q. No. 6, 7, 8, 9, 10, 11, 12, 13

Example 2

$$(x-7)(x-3)(x+1)(x+5) - 1680 = 0$$

$$(x-7)(x-3)(x+1)(x+5) = 1680$$

Re-arranging

$$(x+1)(x-3) \cdot (x+5)(x-7) = 1680$$

$$(x^2 - 3x + x - 3) \cdot (x^2 - 7x + 5x - 35) = 1680$$

$$(x^2 - 2x - 3)(x^2 - 2x - 35) = 1680$$

Put $x^2 - 2x = y$

Then $(y-3)(y-35) = 1680$

$$y^2 - 35y - 3y + 105 = 1680$$

$y^2 - 38y + 105 - 1680 = 0$

$y^2 - 38y - 1575 = 0$

Using $y = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$y = \dfrac{-(-38) \pm \sqrt{(-38)^2 - 4(1)(-1575)}}{2(1)}$

$y = \dfrac{38 \pm \sqrt{1444 + 6300}}{2}$

$y = \dfrac{38 \pm \sqrt{7744}}{2} \Rightarrow y = \dfrac{38 \pm 88}{2}$

$y = \dfrac{38 + 88}{2}$, $y = \dfrac{38 - 88}{2}$

$y = \dfrac{126}{2}$, $y = \dfrac{-50}{2}$

$y = 63$, $y = -25$

If $y = 63$ Then , If $y = -25$ Then

$x^2 - 2x = 63$ $\quad$ $x^2 - 2x = -25$

$x^2 - 2x - 63 = 0$ $\quad$ $x^2 - 2x + 25 = 0$

$x^2 + 7x - 9x - 63 = 0$ $\quad$ Using $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x(x+7) - 9(x+7) = 0$

$(x+7)(x-9) = 0$ $\quad$ $x = \dfrac{-(-2) \pm \sqrt{(-2)^2 - 4 \times 25}}{2(1)}$

$x + 7 = 0,\ x - 9 = 0$

$x = -7,\ x = 9$ $\quad$ $x = \dfrac{2 \pm \sqrt{4 - 100}}{2}$

$x = \dfrac{2 \pm \sqrt{-96}}{2} \Rightarrow x = \dfrac{2 \pm \sqrt{16 \times 6 \times (-1)}}{2}$

$x = \dfrac{2 \pm 4\sqrt{6}\sqrt{-1}}{2} \Rightarrow x = \dfrac{2(1 \pm 2\sqrt{6}\, i)}{2}$

$x = 1 \pm 2\sqrt{6}\, i$ $\quad$ $\because \sqrt{-1} = i$

$\{-7, 9, \ 1 \pm 2\sqrt{6}\, i\}$

Type No.3

When given equation has variable as an exponent, just like Example. 3,4, Q.No.14 15, 16, 17.

Example 3 $2^{2x} - 3 \cdot 2^{x+2} + 32 = 0$

$2^{2x} - 3 \cdot 2^x \cdot 2^2 + 32 = 0$

$(2^x)^2 - 3 \cdot 2^x \cdot 4 + 32 = 0$

$(2^x)^2 - 12 \cdot 2^x + 32 = 0$

put $2^x = y$

Then $y^2 - 12y + 32 = 0$

$32y^2 \to -4y,\ -8y$

$y^2 - 4y - 8y + 32 = 0$

$y(y-4) - 8(y-4) = 0$

$(y-4)(y-8) = 0$

$\Rightarrow y - 4 = 0,\ y - 8 = 0$

$y = 4,\ y = 8$

If $y = 4$ Then $\quad$ If $y = 8$ Then

$2^x = 4$ $\quad$ $2^x = 8$

$2^x = 2^2$ $\quad$ $2^x = 2^3$

$\Rightarrow x = 2$ $\quad$ $\Rightarrow x = 3$

$\{2, 3\}$

Example 4 $4^{1+x} + 4^{1-x} = 10$

$4^1 \cdot 4^x + 4^1 \cdot 4^{-x} = 10$

$4 \cdot 4^x + 4 \cdot \dfrac{1}{4^x} = 10$

put $4^x = y$. Then

$4y + 4 \cdot \dfrac{1}{y} = 10$

$4y^2 + 4 = 10y$

$4y^2 - 10y + 4 = 0$

$2y^2 - 5y + 2 = 0$

Factorizing $2y^2 - y - 4y + 2 = 0$

$y(2y-1) - 2(2y-1) = 0$

$(y-2)(2y-1) = 0$

$y - 2 = 0,\ 2y - 1 = 0$

$\Rightarrow y = 2,\ y = 1/2$

If $y = 2$ Then , If $y = 1/2$ Then

$4^x = 2$ $\quad$ $4^x = 1/2$

$(2^2)^x = 2$ $\quad$ $(2^2)^x = 2^{-1}$

$2^{2x} = 2^1$ $\quad$ $2^{2x} = 2^{-1}$

$\Rightarrow 2x = 1$ and $2x = -1$ $\quad$ $\{1/2, -1/2\}$

$\Rightarrow x = 1/2$ and $x = -1/2$

Type No.4

When given equation remains unchanged by replacing x with $1/x$, such equations are called reciprocal equations such as Example 5, Q. No. 18, 19, 20, 21, 22, 23, 24

Example 5

$$x^4-3x^3+4x^2-3x+1=0$$

Dividing by x^2

$$\frac{x^4}{x^2}-\frac{3x^3}{x^2}+\frac{4x^2}{x^2}-\frac{3x}{x^2}+\frac{1}{x^2}=\frac{0}{x^2}$$

$$x^2-3x+4-\frac{3}{x}+\frac{1}{x^2}=0$$

$$x^2+\frac{1}{x^2}-3x-\frac{3}{x}+4=0$$

$$x^2+\frac{1}{x^2}-3\left(x+\frac{1}{x}\right)+4=0$$

put $x+\frac{1}{x}=y$

$$\Rightarrow \left(x+\frac{1}{x}\right)^2=y^2 \text{ or } x^2+\frac{1}{x^2}+2=y^2$$

$$\Rightarrow x^2+\frac{1}{x^2}=y^2-2$$

Then given equation takes form

$$y^2-2-3y+4=0$$

$$y^2-3y+2=0$$

$$y^2-y-2y+2=0$$

$$2y^2 \to -y, -2y$$

$$y(y-1)-2(y-1)=0$$

$$(y-1)(y-2)=0$$

$$y-1=0,\quad y-2=0$$

$$y=1,\quad y=2$$

If $y=1$ then

$$x+\frac{1}{x}=1$$

$$x^2+1=x$$

$$x^2-x+1=0$$

Using $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

$$x=\frac{-(-1)\pm\sqrt{(-1)^2-4(1)(1)}}{2(1)}$$

$$x=\frac{1\pm\sqrt{1-4}}{2}$$

$$x=\frac{1\pm\sqrt{-3}}{2}$$

If $y=2$ then

$$x+\frac{1}{x}=2$$

$$x^2+1=2x$$

$$x^2-2x+1=0$$

$$(x-1)^2=0$$

$$x-1=0$$

$$x=1$$

$$\left\{1,\ \frac{1\pm\sqrt{-3}}{2}\right\}$$

EXERCISE 4.2

Q.1

$$x^4-6x^2+8=0$$

$$(x^2)^2-6x^2+8=0$$

put $x^2=y$ then

$$y^2-6y+8=0$$

$$y^2-2y-4y+8=0$$

$$8y^2 \to -2y, -4y$$

$$y(y-2)-4(y-2)=0$$

$$(y-2)(y-4)=0$$

$$y-2=0,\quad y-4=0$$

$$y=2,\quad y=4$$

If $y=2$ then

$$x^2=2$$

$$x=\pm\sqrt{2}$$

If $y=4$ then

$$x^2=4$$

$$x=\pm\sqrt{4}$$

$$x=\pm 2$$

$$\{\pm\sqrt{2},\ \pm 2\}$$

Q.2

$$x^{-2}-10=3x^{-1}$$

$$x^{-2}-3x^{-1}-10=0$$

$$(x^{-1})^2-3x^{-1}-10=0$$

put $x^{-1}=y$ then

$$y^2-3y-10=0$$

$$y^2+2y-5y-10=0$$

$$-10y^2 \to 2y, -5y$$

$$y(y+2)-5(y+2)=0$$

$$(y-5)(y+2)=0$$

$$y-5=0,\quad y+2=0$$

$$y=5,\quad y=-2$$

If $y=5$ then

$$x^{-1}=5$$

$$\frac{1}{x}=5$$

$$\Rightarrow x=1/5$$

If $y=-2$ then

$$x^{-1}=-2$$

$$\frac{1}{x}=-2$$

$$x=-1/2$$

$$\{1/5,\ -1/2\}$$

Q.3

$$x^6-9x^3+8=0$$

$$(x^3)^2-9x^3+8=0$$

put $x^3=y$ then

$$y^2-9y+8=0$$

$$y^2-y-8y+8=0$$

$$y(y-1)-8(y-1)=0$$

$$(y-1)(y-8)=0$$

$$y-1=0\ ,\ y-8=0$$

$$y=1\ ,\ y=8$$

$8y^2$ → $-y$, $-8y$

If $y=1$ Then

$$x^3=1$$

$$x^3-1=0$$

$$x^3-(1)^3=0$$

If $y=8$ Then

$$x^3=8$$

$$x^3-8=0$$

$$x^3-(2)^3=0$$

$\because a^3-b^3=(a-b)(a^2+ab+b^2)$

So

$$(x-1)(x^2+x+1)=0$$

$$x-1=0,\ x^2+x+1=0$$

$$x=1,\ x^2+x+1=0$$

Using $x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$

$$x=\frac{-1\pm\sqrt{(1)^2-4(1)(1)}}{2(1)}$$

$$x=\frac{-1\pm\sqrt{1-4}}{2}$$

$$x=\frac{-1\pm\sqrt{-3}}{2}$$

$$(x-2)(x^2+2x+4)=0$$

$$x-2=0,\ x^2+2x+4=0$$

$$x=2\ ,\ x^2+2x+4=0$$

$$x=\frac{-2\pm\sqrt{(2)^2-4(1)(4)}}{2(1)}$$

$$x=\frac{-2\pm\sqrt{4-16}}{2}$$

$$x=\frac{-2\pm\sqrt{-12}}{2}$$

$$x=\frac{-2\pm\sqrt{4}\sqrt{-3}}{2}$$

$$x=\frac{-2\pm 2\sqrt{-3}}{2}$$

$$x=\frac{2(-1\pm\sqrt{-3})}{2}$$

$$x=-1\pm\sqrt{-3}$$

$\left\{1, 2, \dfrac{-1\pm\sqrt{-3}}{2}, -1\pm\sqrt{-3}\right\}$.

Q.4

$$8x^6-19x^3-27=0$$

$$8(x^3)^2-19x^3-27=0$$

put $x^3=y$ Then

$$8y^2-19y-27=0$$

$$8y^2+8y-27y-27=0$$

$$8y(y+1)-27(y+1)=0$$

$$(y+1)(8y-27)=0$$

$$y+1=0,\ 8y-27=0$$

$$y=-1\ ,\ y=27/8$$

$-216y^2$ → $8y$, $-27y$

If $y=-1$ Then

$$x^3=-1$$

$$x^3+1=0$$

$$x^3+(1)^3=0$$

$$(x)^3+(1)^3=0$$

If $y=27/8$ then

$$x^3=27/8$$

$$8x^3=27$$

$$8x^3-27=0$$

$$(2x)^3-(3)^3=0$$

$\because a^3+b^3=(a+b)(a^2-ab+b^2)$

& $a^3-b^3=(a-b)(a^2+ab+b^2)$

$$(x+1)(x^2-x+1)=0$$

Either $x+1=0$

$\Rightarrow x=-1$

OR $x^2-x+1=0$

$$(2x-3)(4x^2+6x+9)=0$$

Either $2x-3=0$

$\Rightarrow x=3/2$

OR $4x^2+6x+9=0$

Using $x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$

$$x=\frac{-(-1)\pm\sqrt{(-1)^2-4(1)(1)}}{2(1)}$$

$$x=\frac{1\pm\sqrt{1-4}}{2}$$

$$x=\frac{1\pm\sqrt{-3}}{2}$$

$$x=\frac{-6\pm\sqrt{(6)^2-4(4)(9)}}{2(4)}$$

$$x=\frac{-6\pm\sqrt{36-144}}{8}$$

$$x=\frac{-6\pm\sqrt{-108}}{8}$$

$$x=\frac{-6\pm\sqrt{36\times(-3)}}{8}$$

$$x=\frac{-6\pm 6\sqrt{-3}}{8}$$

$$x=6\left(\frac{-1\pm\sqrt{-3}}{8}\right)\rightarrow x=3\left(\frac{-1\pm\sqrt{-3}}{4}\right)$$

$\left\{-1, 3/2, \dfrac{1\pm\sqrt{-3}}{2}, 3\left(\dfrac{-1\pm\sqrt{-3}}{4}\right)\right\}$

Q.5 $x^{2/5}+8=6x^{1/5}$

$x^{2/5}-6x^{1/5}+8=0$

$(x^{1/5})^2-6x^{1/5}+8=0$

put $x^{1/5}=y$ Then

$y^2-6y+8=0$

$y^2-2y-4y+8=0$ $\quad$ $8y^2 \to -2y,\ -4y$

$y(y-2)-4(y-2)=0$

$(y-2)(y-4)=0$

$y-2=0\ ,\ y-4=0$

If $y=2$ Then $\quad$ If $y=4$ Then

$x^{1/5}=2$ $\quad$ $x^{1/5}=4$

$(x^{1/5})^5=2^5$ $\quad$ $(x^{1/5})^5=4^5$

$x=32$ $\quad$ $x=1024$

$\{32, 1024\}$

Q6 $(x+1)(x+2)(x+3)(x+4)=24$

Re-arranging it

$(x+1)(x+4)\cdot(x+2)(x+3)=24$

$(x^2+4x+x+4)\cdot(x^2+3x+2x+6)=24$

$(x^2+5x+4)\cdot(x^2+5x+6)=24$

put $x^2+5x=y$ Then

$(y+4)(y+6)=24$

$y^2+6y+4y+24=24$

$y^2+10y+24-24=0$

$y^2+10y=0$

$y(y+10)=0$

$y=0\ ,\ y+10=0 \Rightarrow y=-10$

If $y=0$ Then

$x^2+5x=0$

$x(x+5)=0$

$x=0,\ x+5=0$

$x=-5$

If $y=-10$ Then

$x^2+5x=-10$

$x^2+5x+10=0$

Using $x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$

$$x=\frac{-5\pm\sqrt{(5)^2-4(1)(10)}}{2(1)}$$

$$x=\frac{-5\pm\sqrt{25-40}}{2} \Rightarrow x=\frac{-5\pm\sqrt{-15}}{2}$$

$$\left\{0,\ -5,\ \frac{-5\pm\sqrt{-15}}{2}\right\}$$

Q.7 $(x-1)(x+5)(x+8)(x+2)-880=0$

$(x-1)(x+5)(x+8)(x+2)=880$

Re-arranging it

$(x-1)(x+8)\cdot(x+2)(x+5)=880$

$(x^2+8x-x-8)\cdot(x^2+5x+2x+10)=880$

$(x^2+7x-8)\cdot(x^2+7x+10)=880$

put $x^2+7x=y$ Then

$(y-8)(y+10)=880$

$y^2+10y-8y-80=880$

$y^2+2y-80-880=0$

$y^2+2y-960=0$

$y^2+32y-30y-960=0$ $\quad$ $-960y^2 \to 32y,\ -30y$

$y(y+32)-30(y+32)=0$

$(y-30)(y+32)=0$

$y-30=0,\ y+32=0$

$y=30\ ,\ y=-32$

If $y=30$ Then $\quad$ If $y=-32$ Then

$x^2+7x=30\ ,\quad x^2+7x=-32$

$x^2+7x-30=0\ ,\quad x^2+7x+32=0$

$x^2-3x+10x-30=0,\quad$ Using $x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$

$x(x-3)+10(x-3)=0,$

$(x-3)(x+10)=0$

$x-3=0,\ x+10=0$

$x=3,\ x=-10$

$x=\dfrac{-7\pm\sqrt{(7)^2-4(1)(32)}}{2(1)}$

$x=\dfrac{-7\pm\sqrt{49-128}}{2}$

$x=\dfrac{-7\pm\sqrt{-79}}{2}$

$\left\{3,\ -10,\ \dfrac{-7\pm\sqrt{-79}}{2}\right\}$

Q.8 $(x-5)(x-7)(x+6)(x+4)-504=0$

$(x-5)(x-7)(x+6)(x+4)=504$

Re-arranging it

$(x-5)(x+4)\cdot(x-7)(x+6)=504$

$(x^2+4x-5x-20)\cdot(x^2+6x-7x-42)=504$

$(x^2-x-20)\cdot(x^2-x-42)=504$

put $x^2-x=y$ Then

$(y-20)(y-42)=504$

$y^2-42y-20y+840-504=0$

$y^2-62y+336=0$

$y^2-6y-56y+336=0$

$336y^2$

$-6y \quad -56y$

$y(y-6)-56(y-6)=0$

$(y-6)(y-56)=0$

$y-6=0,\ y-56=0$

$y=6,\ y=56$

If $y=6$ Then , If $y=56$ Then

$x^2-x=6 \qquad x^2-x=56$

$x^2-x-6=0 \qquad x^2-x-56=0$

$x^2+2x-3x-6=0 \qquad x^2+7x-8x-56=0$

$x(x+2)-3(x+2)=0 \qquad x(x+7)-8(x+7)=0$

$(x-3)(x+2)=0 \qquad (x-8)(x+7)=0$

$x-3=0,\ x+2=0 \qquad x-8=0,\ x+7=0$

$x=3,\ x=-2 \qquad x=8,\ x=-7$

$\{3,-2,8,-7\}$

Q.9 $(x-1)(x-2)(x-8)(x+5)+360=0$

$(x-1)(x-2)(x-8)(x+5)=-360$

Re-arranging it

$(x-1)(x-2)\cdot(x-8)(x+5)=-360$

$(x^2-2x-x+2)\cdot(x^2+5x-8x-40)=-360$

$(x^2-3x+2)\cdot(x^2-3x-40)=-360$

put $x^2-3x=y$ Then

$(y+2)(y-40)=-360$

$y^2-40y+2y-80+360=0$

$y^2-38y+280=0$

$y^2-10y-28y+280=0$

$280y^2$

$-10y \quad -28y$

$y(y-10)-28(y-10)=0$

$(y-10)(y-28)=0$

$y-10=0,\ y-28=0$

$y=10,\ y=28$

If $y=10$ Then , If $y=28$ Then

$x^2-3x=10 \qquad x^2-3x=28$

$x^2-3x-10=0 \qquad x^2-3x-28=0$

$x^2+2x-5x-10=0 \qquad x^2+4x-7x-28=0$

$x(x+2)-5(x+2)=0 \qquad x(x+4)-7(x+4)=0$

$(x-5)(x+2)=0 \qquad (x-7)(x+4)=0$

$x-5=0,\ x+2=0 \qquad x-7=0,\ x+4=0$

$x=5,\ x=-2 \qquad x=7,\ x=-4$

$\{5,-2,7,-4\}$.

Q.10 $(x+1)(2x+3)(2x+5)(x+3)=945$

Re-arranging it

$(x+1)(x+3)\cdot(2x+3)(2x+5)=945$

$(x^2+3x+x+3)\cdot(4x^2+10x+6x+15)=945$

$(x^2+4x+3)\cdot(4x^2+16x+15)=945$

$(x^2+4x+3)\cdot(4(x^2+4x)+15)=945$

put $x^2+4x=y$ Then

$$(y+3)(4y+15)=945$$
$$4y^2+15y+12y+45-945=0$$
$$4y^2+27y-900=0$$
$$4y^2-48y+75y-900=0 \qquad -3600y^2 \to -48y,\ 75y$$
$$4y(y-12)+75(y-12)=0$$
$$(y-12)(4y+75)=0$$
$$y-12=0,\quad 4y+75=0$$
$$y=12,\quad y=\frac{-75}{4}$$

If $y=12$ then, If $y=\frac{-75}{4}$ Then

$$x^2+4x=12,\quad x^2+4x=\frac{-75}{4}$$
$$x^2+4x-12=0,\quad 4x^2+16x=-75$$
$$x^2+4x-12=0,\quad 4x^2+16x+75=0$$
$$x^2-2x+6x-12=0,\quad \text{using } x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$
$$x(x-2)+6(x-2)=0$$
$$(x-2)(x+6)=0 \qquad x=\frac{-16\pm\sqrt{(16)^2-4(4)(75)}}{2(4)}$$
$$x-2=0,\quad x+6=0$$
$$x=2,\quad x=-6 \qquad x=\frac{-16\pm\sqrt{256-1200}}{8}$$
$$x=\frac{-16\pm\sqrt{-944}}{8} \Rightarrow x=\frac{-16\pm\sqrt{16\times(-59)}}{8}$$
$$x=\frac{-16\pm4\sqrt{-59}}{8} \Rightarrow x=\frac{4(-4\pm\sqrt{-59})}{8}$$
$$x=\frac{-4\pm\sqrt{-59}}{2} \Rightarrow x=\frac{-4\pm\sqrt{59}\sqrt{-1}}{2}$$
$$x=\frac{-4\pm\sqrt{59}\,i}{2} \qquad \because \sqrt{-1}=i \text{ (iota)}$$
$$\left\{-6,\ 2,\ \frac{-4\pm\sqrt{59}\,i}{2}\right\}$$

Q.11 $(2x-7)(x^2-9)(2x+5)-91=0$

$$(2x-7)(x-3)(x+3)\cdot(2x+5)=91$$

Re-arranging it

$$(2x-7)(x+3)\cdot(x-3)(2x+5)=91$$
$$(2x^2+6x-7x-21)\cdot(2x^2+5x-6x-15)=91$$
$$(2x^2-x-21)\cdot(2x^2-x-15)=91$$

put $2x^2-x=y$ then

$$(y-21)(y-15)=91$$
$$y^2-15y-21y+315-91=0$$
$$y^2-36y+224=0$$
$$y^2-8y-28y+224=0 \qquad 224y^2 \to -8y,\ -28y$$
$$y(y-8)-28(y-8)=0$$
$$(y-8)(y-28)=0$$
$$y-8=0,\quad y-28=0$$
$$y=8,\quad y=28$$

If $y=8$ then, If $y=28$ then

$$2x^2-x=8,\quad 2x^2-x=28$$
$$2x^2-x-8=0,\quad 2x^2-x-28=0$$

using $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

$$x=\frac{-(-1)\pm\sqrt{(-1)^2-4(2)(-8)}}{2(2)}$$
$$x=\frac{1\pm\sqrt{1+64}}{4}$$
$$x=\frac{1\pm\sqrt{65}}{4}$$

$$2x^2-8x+7x-28=0$$
$$2x(x-4)+7(x-4)=0$$
$$(x-4)(2x+7)=0$$
$$x-4=0,\quad 2x+7=0$$
$$x=4,\quad x=-\frac{7}{2}$$

$$\left\{4,\ -\frac{7}{2},\ \frac{1\pm\sqrt{65}}{4}\right\}$$

Q.12 $(x^2+6x+8)(x^2+14x+48)=105$

Factorizing it

$$(x^2+2x+4x+8)\cdot(x^2+6x+8x+48)=105$$
$$\left[x(x+2)+4(x+2)\right]\left[x(x+6)+8(x+6)\right]=105$$
$$(x+2)(x+4)\cdot(x+6)(x+8)=105$$

Re-arranging it

$$(x+2)(x+8)\cdot(x+4)(x+6)=105$$
$$(x^2+8x+2x+16)\cdot(x^2+6x+4x+24)=105$$
$$(x^2+10x+16)\cdot(x^2+10x+24)=105$$

put $x^2+10x=y$ Then

$(y+16)(y+24)=105$

$y^2+24y+16y+384-105=0$

$y^2+40y+279=0$

$y^2+9y+31y+279=0$

$279y^2$

$9y \quad 31y$

$y(y+9)+31(y+9)=0$

$(y+9)(y+31)=0$

$y+9=0\ ,\quad y+31=0$

$y=-9\ ,\quad y=-31$

If $y=-9$ Then ,

$x^2+10x=-9$

$x^2+10x+9=0$

$x^2+x+9x+9=0$

$x(x+1)+9(x+1)=0$

$(x+1)(x+9)=0$

$x+1=0,\ x+9=0$

$x=-1;\ x=-9$

If $y=-31$ Then

$x^2+10x=-31$

$x^2+10x+31=0$

Using $x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$

$x=\dfrac{-10\pm\sqrt{(10)^2-4(1)(31)}}{2(1)}$

$x=\dfrac{-10\pm\sqrt{100-124}}{2}$

$x=\dfrac{-10\pm\sqrt{-24}}{2} \Rightarrow x=\dfrac{-10\pm\sqrt{4(-6)}}{2}$

$x=\dfrac{-10\pm2\sqrt{-6}}{2} \Rightarrow x=\dfrac{2(-5\pm\sqrt{-6})}{2}$

$x=-5\pm\sqrt{-6} \Rightarrow x=-5\pm\sqrt{6}\sqrt{-1}$

$x=-5\pm\sqrt{6}\,i \qquad \because \sqrt{-1}=i$

$\{-1,\ -9,\ -5\pm\sqrt{-6}\}$

OR $\{-1,\ -9,\ -5\pm\sqrt{6}\,i\}$

Q.13 $(x^2+6x-27)(x^2-2x-35)=385$

Factorizing it

$(x^2-3x+9x-27)\cdot(x^2+5x-7x-35)=385$

$[x(x-3)+9(x-3)]\,[x(x+5)-7(x+5)]=385$

$(x-3)(x+9)\cdot(x-7)(x+5)=385$

Re-arranging it

$(x-3)(x+5)\cdot(x-7)(x+9)=385$

$(x^2+5x-3x-15)\cdot(x^2+9x-7x-63)=385$

$(x^2+2x-15)\cdot(x^2+2x-63)=385$

put $x^2+2x=y$ Then

$(y-15)(y-63)=385$

$y^2-63y-15y+945-385=0$

$y^2-78y+560=0$

$y^2-8y-70y+560=0$

$560y^2$

$-8y \quad -70y$

$y(y-8)-70(y-8)=0$

$(y-8)(y-70)=0$

$y-8=0,\ y-70=0$

$y=8\ ,\quad y=70$

If $y=8$ Then

$x^2+2x=8$;

$x^2+2x-8=0$,

$x^2-2x+4x-8=0$

$x(x-2)+4(x-2)=0$

$(x-2)(x+4)=0$

$x-2=0,\ x+4=0$

$x=2\ ,\ x=-4$

If $y=70$ Then

$x^2+2x=70$

$x^2+2x-70=0$

Using $x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$

$x=\dfrac{-2\pm\sqrt{(2)^2-4(1)(-70)}}{2(1)}$

$x=\dfrac{-2\pm\sqrt{4+280}}{2}$

$x=\dfrac{-2\pm\sqrt{284}}{2} \rightarrow x=\dfrac{-2\pm\sqrt{4\times71}}{2}$

$x=\dfrac{-2\pm2\sqrt{71}}{2} \rightarrow x=\dfrac{2(-1\pm\sqrt{71})}{2}$

$x=-1\pm\sqrt{71} \qquad \{2,-4,-1\pm\sqrt{71}\}$

Q.14 $4\cdot2^{2x+1}-9\cdot2^{x}+1=0$

$4\cdot2^{2x}\cdot2^{1}-9\cdot2^{x}+1=0$

$4\cdot(2^{x})^{2}\cdot2-9\cdot2^{x}+1=0$

$8\cdot(2^{x})^{2}-9\cdot2^{x}+1=0$

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put $2^x = y$ Then

$8y^2 - 9y + 1 = 0$

$8y^2 - 8y - y + 1 = 0$

$8y(y-1) - 1(y-1) = 0$

$(y-1)(8y-1) = 0$

$y-1 = 0, \quad 8y-1 = 0$

$y = 1, \quad y = 1/8$

If $y = 1$ Then, If $y = 1/8$ Then

$2^x = 1, \quad 2^x = 1/8$

$2^x = 2^0, \quad 2^x = \frac{1}{2^3}$

$2^x = 2^0, \quad 2^x = 2^{-3}$

$\Rightarrow x = 0, \quad x = -3$

$\{0, -3\}$

Q.15 $2^x + 2^{-x+6} - 20 = 0$

$2^x + 2^{-x} \cdot 2^6 - 20 = 0$

$2^x + 2^{-x} \cdot 64 - 20 = 0$

$2^x + \frac{64}{2^x} - 20 = 0$

put $2^x = y$ Then

$y + \frac{64}{y} - 20 = 0$

$y^2 + 64 - 20y = 0$

$y^2 - 20y + 64 = 0$

$y^2 - 4y - 16y + 64 = 0$

$y(y-4) - 16(y-4) = 0$

$(y-4)(y-16) = 0$

$y-4 = 0, \quad y-16 = 0$

$y = 4, \quad y = 16$

If $y = 4$ Then If $y = 16$ then

$2^x = 4, \quad 2^x = 16$

$2^x = 2^2 \quad 2^x = 2^4$

$\Rightarrow x = 2 \quad x = 4$

$\{2, 4\}$

Q.16 $4^x - 3 \cdot 2^{x+3} + 128 = 0$

$(2^2)^x - 3 \cdot 2^x \cdot 2^3 + 128 = 0$

$(2^x)^2 - 3 \cdot 2^x \cdot 8 + 128 = 0$

$(2^x)^2 - 24 \cdot 2^x + 128 = 0$

put $2^x = y$ Then

$y^2 - 24y + 128 = 0$

$y^2 - 8y - 16y + 128 = 0$

$y(y-8) - 16(y-8) = 0$

$(y-8)(y-16) = 0$

$y-8 = 0, \quad y-16 = 0$

$y = 8, \quad y = 16$

If $y = 8$ Then, If $y = 16$ Then

$2^x = 8, \quad 2^x = 16$

$2^x = 2^3, \quad 2^x = 2^4$

$\rightarrow x = 3, \quad x = 4$

$\{3, 4\}$

Q.17 $3^{2x-1} - 12 \cdot 3^x + 81 = 0$

$3^{2x} \cdot 3^{-1} - 12 \cdot 3^x + 81 = 0$

$(3^x)^2 \cdot \frac{1}{3} - 12 \cdot 3^x + 81 = 0$

Multiplying by 3

$(3^x)^2 - 36 \cdot 3^x + 243 = 0$

put $3^x = y$ Then

$y^2 - 36y + 243 = 0$

$y^2 - 9y - 27y + 243 = 0$

$y(y-9) - 27(y-9) = 0$

$(y-9)(y-27) = 0$

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$y-9=0, \quad y-27=0$

$y=9, \quad y=27$

If $y=9$ Then, If $y=27$ Then

$3^x=9, \quad 3^x=27$

$3^x=3^2, \quad 3^x=3^3$

$\Rightarrow x=2, \quad x=3$

$\{2,3\}$

Q.18 $\left(x+\frac{1}{x}\right)^2-3\left(x+\frac{1}{x}\right)-4=0$

put $x+\frac{1}{x}=y$ Then

$y^2-3y-4=0$

$y^2+y-4y-4=0$

$y(y+1)-4(y+1)=0$

$(y+1)(y-4)=0$

$y+1=0, \quad y-4=0$

$y=-1, \quad y=4$

If $y=-1$ Then, If $y=4$ Then

$x+\frac{1}{x}=-1 \qquad x+\frac{1}{x}=4$

$x^2+1=-x \qquad x^2+1=4x$

$x^2+x+1=0 \qquad x^2-4x+1=0$

Using $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

$x=\frac{-1\pm\sqrt{(1)^2-4(1)(1)}}{2(1)}$

$x=\frac{-1\pm\sqrt{1-4}}{2}$

$x=\frac{-1\pm\sqrt{-3}}{2}$

$x=\frac{-1\pm\sqrt{3}\sqrt{-1}}{2}$

$x=\frac{-1\pm\sqrt{3}\,i}{2}$

$x=\frac{-(-4)\pm\sqrt{(-4)^2-4(1)(1)}}{2(1)}$

$x=\frac{4\pm\sqrt{16-4}}{2}$

$x=\frac{4\pm\sqrt{12}}{2}$

$x=\frac{4\pm\sqrt{4\times 3}}{2}$

$x=\frac{4\pm 2\sqrt{3}}{2}=2\pm\sqrt{3}$

$\left\{\frac{-1\pm\sqrt{3}\,i}{2},\ 2\pm\sqrt{3}\right\}$

Q.19 $x^2+x-4+\frac{1}{x}+\frac{1}{x^2}=0$

$x^2+\frac{1}{x^2}+x+\frac{1}{x}-4=0$

put $x+\frac{1}{x}=y$ Then

$\left(x+\frac{1}{x}\right)^2=y^2$

$x^2+\frac{1}{x^2}+2=y^2 \Rightarrow x^2+\frac{1}{x^2}=y^2-2$

Given equation takes form

$y^2-2+y-4=0$

$y^2+y-6=0$

$y^2-2y+3y-6=0$

$y(y-2)+3(y-2)=0$

$(y-2)(y+3)=0$

$y-2=0, \quad y+3=0$

$y=2, \quad y=-3$

If $y=2$ then, If $y=-3$ Then.

$x+\frac{1}{x}=2, \qquad x+\frac{1}{x}=-3$

$x^2+1=2x, \qquad x^2+1=-3x$

$x^2-2x+1=0, \qquad x^2+3x+1=0$

$(x-1)^2=0 \qquad$ Using $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

$x-1=0$

$\Rightarrow x=1 \qquad x=\frac{-3\pm\sqrt{(3)^2-4(1)(1)}}{2(1)}$

$x=\frac{-3\pm\sqrt{9-4}}{2} \Rightarrow x=\frac{-3\pm\sqrt{5}}{2}$

$\left\{1,\ \frac{-3\pm\sqrt{5}}{2}\right\}$

Q.20 $\left(x-\frac{1}{x}\right)^2+3\left(x+\frac{1}{x}\right)=0$

$x^2+\frac{1}{x^2}-2+3\left(x+\frac{1}{x}\right)=0$

put $x+\frac{1}{x}=y$

$\Rightarrow \left(x+\frac{1}{x}\right)^2=y^2$

$x^2+\frac{1}{x^2}+2=y^2 \Rightarrow x^2+\frac{1}{x^2}=y^2-2$

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Then given equation takes form

$$y^2-2-2+3y=0$$
$$y^2+3y-4=0$$
$$y^2-y+4y-4=0$$
$$y(y-1)+4(y-1)=0$$
$$(y-1)(y+4)=0$$
$$y-1=0,\quad y+4=0$$
$$y=1,\quad y=-4$$

If $y=1$ then , If $y=-4$ then

$$x+\frac{1}{x}=1,\quad x+\frac{1}{x}=-4$$
$$x^2+1=x,\quad x^2+1=-4x$$
$$x^2-x+1=0,\quad x^2+4x+1=0$$

Using $x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$

$$x=\frac{-(-1)\pm\sqrt{(-1)^2-4(1)(1)}}{2(1)}$$
$$x=\frac{1\pm\sqrt{1-4}}{2}$$
$$x=\frac{1\pm\sqrt{-3}}{2}$$
$$x=\frac{1\pm\sqrt{3}\sqrt{-1}}{2}$$
$$x=\frac{1\pm\sqrt{3}\,i}{2}$$

$$x=\frac{-4\pm\sqrt{(4)^2-4(1)(1)}}{2(1)}$$
$$x=\frac{-4\pm\sqrt{16-4}}{2}$$
$$x=\frac{-4\pm\sqrt{12}}{2}$$
$$x=\frac{-4\pm\sqrt{4\times3}}{2}$$
$$x=\frac{-4\pm2\sqrt{3}}{2}$$
$$x=\frac{2(-2\pm\sqrt{3})}{2}$$
$$x=-2\pm\sqrt{3}$$

$$\left\{\frac{1\pm\sqrt{-3}}{2},\ -2\pm\sqrt{3}\right\}$$

OR.

$$\left\{\frac{1\pm\sqrt{3}\,i}{2},\ -2\pm\sqrt{3}\right\}$$

Q.21 $2x^4-3x^3-x^2-3x+2=0$

Dividing by x^2

$$\frac{2x^4}{x^2}-\frac{3x^3}{x^2}-\frac{x^2}{x^2}-\frac{3x}{x^2}+\frac{2}{x^2}=\frac{0}{x^2}$$
$$2x^2-3x-1-\frac{3}{x}+\frac{2}{x^2}=0$$
$$2x^2+\frac{2}{x^2}-3x-\frac{3}{x}-1=0$$
$$2\left(x^2+\frac{1}{x^2}\right)-3\left(x+\frac{1}{x}\right)-1=0$$

put $x+\dfrac{1}{x}=y$

$$\Rightarrow \left(x+\frac{1}{x}\right)^2=y^2$$
$$x^2+\frac{1}{x^2}+2=y^2 \Rightarrow x^2+\frac{1}{x^2}=y^2-2$$

The given equation takes form

$$2(y^2-2)-3y-1=0$$
$$2y^2-4-3y-1=0$$
$$2y^2-3y-5=0$$
$$2y^2+2y-5y-5=0$$
$$2y(y+1)-5(y+1)=0$$
$$(y+1)(2y-5)=0$$
$$y+1=0,\quad 2y-5=0\Rightarrow y=5/2$$
$$y=-1,\quad y=5/2$$

If $y=-1$ Then , If $y=5/2$ then

$$x+\frac{1}{x}=-1,\quad x+\frac{1}{x}=5/2$$
$$x^2+1=-x,\quad 2x^2+2=5x$$
$$x^2+x+1=0,\quad 2x^2-5x+2=0$$

$$2x^2-x-4x+2=0$$
$$x(2x-1)-2(2x-1)=0$$
$$(x-2)(2x-1)=0$$
$$x-2=0,\quad 2x-1=0$$
$$x=2,\quad x=1/2$$

Using

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$
$$x=\frac{-1\pm\sqrt{(1)^2-4(1)(1)}}{2(1)}$$
$$x=\frac{-1\pm\sqrt{1-4}}{2}$$
$$x=\frac{-1\pm\sqrt{-3}}{2}\Rightarrow x=\frac{-1\pm\sqrt{3}\,i}{2}$$

$$\left\{2,\tfrac{1}{2},\frac{-1\pm\sqrt{-3}}{2}\right\} \text{ OR. } \left\{2,\tfrac{1}{2},\frac{-1\pm\sqrt{3}\,i}{2}\right\}$$

Q.22 $2x^4+3x^3-4x^2-3x+2=0$

Dividing by x^2

$$\frac{2x^4}{x^2}+\frac{3x^3}{x^2}-\frac{4x^2}{x^2}-\frac{3x}{x^2}+\frac{2}{x^2}=\frac{0}{x^2}$$

$$2x^2+3x-4-\frac{3}{x}+\frac{2}{x^2}=0$$

$$2x^2+\frac{2}{x^2}+3x-\frac{3}{x}-4=0$$

$$2\left(x^2+\frac{1}{x^2}\right)+3\left(x-\frac{1}{x}\right)-4=0$$

put $x-\frac{1}{x}=y$

$$\Rightarrow \left(x-\frac{1}{x}\right)^2=y^2 \Rightarrow x^2+\frac{1}{x^2}-2=y^2$$

$$x^2+\frac{1}{x^2}=y^2+2$$

Given equation takes form

$$2(y^2+2)+3y-4=0$$

$$2y^2+4+3y-4=0$$

$$2y^2+3y=0$$

$$y(2y+3)=0$$

$y=0,\quad 2y+3=0$

$y=0,\quad y=-3/2$

If $y=0$ then $\qquad$ If $y=-3/2$ then

$x-\frac{1}{x}=0 \quad , \quad x-\frac{1}{x}=-3/2$

$x^2-1=0 \qquad 2x^2-2=-3x$

$(x-1)(x+1)=0 \qquad 2x^2+3x-2=0$

$x-1=0, x+1=0 \qquad 2x^2-x+4x-2=0$

$x=1,\ x=-1 \qquad x(2x-1)+2(2x-1)=0$

$$(2x-1)(x+2)=0$$

$2x-1=0,\quad x+2=0$

$x=1/2\ ,\quad x=-2$

$$\left\{1,-1,\tfrac{1}{2},-2\right\}$$

Q.23 $6x^4-35x^3+62x^2-35x+6=0$

Dividing by x^2

$$\frac{6x^4}{x^2}-\frac{35x^3}{x^2}+\frac{62x^2}{x^2}-\frac{35x}{x^2}+\frac{6}{x^2}=\frac{0}{x^2}$$

$$6x^2-35x+62-\frac{35}{x}+\frac{6}{x^2}=0$$

$$6x^2+\frac{6}{x^2}-35x-\frac{35}{x}+62=0$$

$$6\left(x^2+\frac{1}{x^2}\right)-35\left(x+\frac{1}{x}\right)+62=0$$

put $x+\frac{1}{x}=y$

$$\Rightarrow \left(x+\frac{1}{x}\right)^2=y^2$$

$$x^2+\frac{1}{x^2}+2=y^2 \Rightarrow x^2+\frac{1}{x^2}=y^2-2$$

Given equation takes form

$$6(y^2-2)-35y+62=0$$

$$6y^2-12-35y+62=0$$

$$6y^2-35y+50=0$$

$$6y^2-15y-20y+50=0$$

$$3y(2y-5)-10(2y-5)=0$$

$$(2y-5)(3y-10)=0$$

$2y-5=0,\quad 3y-10=0$

$y=5/2\ ,\quad y=10/3$

If $y=5/2$ then , If $y=10/3$ then

$x+\frac{1}{x}=5/2 \quad , \quad x+\frac{1}{x}=10/3$

$2x^2+2=5x \qquad 3x^2+3=10x$

$2x^2-5x+2=0 \qquad 3x^2-10x+3=0$

$2x^2-x-4x+2=0 \qquad 3x^2-x-9x+3=0$

$x(2x-1)-2(2x-1)=0 \qquad x(3x-1)-3(3x-1)=0$

$(2x-1)(x-2)=0 \qquad (3x-1)(x-3)=0$

$2x-1=0,\ x-2=0 \qquad 3x-1=0,\ x-3=0$

$x=1/2\ ,\ x=2 \qquad x=1/3\ ,\ x=3$

$$\left\{\tfrac{1}{2},2,\tfrac{1}{3},3\right\}$$

Q.24 $x^4-6x^2+10-\frac{6}{x^2}+\frac{1}{x^4}=0$

$x^4+\frac{1}{x^4}-6x^2-\frac{6}{x^2}+10=0$

$x^4+\frac{1}{x^4}-6\left(x^2+\frac{1}{x^2}\right)+10=0$

put $x^2+\frac{1}{x^2}=y$

$\Rightarrow \left(x^2+\frac{1}{x^2}\right)^2=y^2$

$x^4+\frac{1}{x^4}+2=y^2 \Rightarrow x^4+\frac{1}{x^4}=y^2-2$

Given equation takes form.

$y^2-2-6y+10=0$

$y^2-6y+8=0$

$y^2-2y-4y+8=0$

$y(y-2)-4(y-2)=0$

$(y-2)(y-4)=0$

$y-2=0,\quad y-4=0$

$y=2,\quad y=4$

If $y=2$ then

$x^2+\frac{1}{x^2}=2$

$x^4+1=2x^2$

$x^4-2x^2+1=0$

$(x^2-1)^2=0$

$x^2-1=0$

$(x-1)(x+1)=0$

$x-1=0,\ x+1=0$

$x=1,\ x=-1$

If $y=4$ then

$x^2+\frac{1}{x^2}=4$

$x^4+1=4x^2$

$x^4-4x^2+1=0$

$(x^2)^2-4x^2+1=0$

Let $x^2=z$ then

$z^2-4z+1=0$

$z=\frac{-(-4)\pm\sqrt{(-4)^2-4(1)(1)}}{2(1)}$

$z=\frac{4\pm\sqrt{16-4}}{2} \Rightarrow z=\frac{4\pm\sqrt{12}}{2}$

$z=\frac{4\pm\sqrt{4\times3}}{2} \Rightarrow z=\frac{4\pm2\sqrt{3}}{2}=\frac{2(2\pm\sqrt{3})}{2}$

$z=2\pm\sqrt{3}$ so $x^2=2\pm\sqrt{3}$

$\Rightarrow x=\pm\sqrt{2\pm\sqrt{3}}$

$\left\{\mathbf{1},-\mathbf{1},\ \pm\sqrt{2\pm\sqrt{3}}\right\}$

Redical Another word used for root, The sign $\sqrt{\ }$ is called redical sign. A number placed to the left of the sign shows the type of root eg. $\sqrt[2]{\ }, \sqrt[3]{\ }, \sqrt[4]{\ }, \ldots\ldots, \sqrt[n]{\ }$ denote Square root, Cube roots, Fourth root, ..., nth roots respectively. Here numbers 2, 3, 4 and n are called index. If there is no number to the left of the sign $\sqrt{\ }$, then root is called square root.

Redical Equations The equations involving redical expressions of the variable are called redical equations.

For example; $\sqrt{2x+8}+\sqrt{x+5}=7$

$3x^2+2x-\sqrt{3x^2+2x-1}-7=0$

$\sqrt{x^2+2x-3}+\sqrt{x^2+7x-8}=\sqrt{5x^2+15x-20}$

are redical equations.

Extraneous Root A number obtained in the process of solving an equation, which is actually not a root of the given equation. In other words, we may say that, a root that does not satisfy given equation is called an extraneous root.

★ There are some types of Redical Equations

Type No. 1

The equation of the form

$l(ax^2+bx)+m\sqrt{ax^2+bx+c}=0$

Questions of this type are

Example 1, Q. No. 1, 2, 10

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