Chapter No 4 Quadratic Equations

Quadratic Equation

An equation containing one or more terms in which the variable is raised to maximum positive power two. In general;

$$ax^2+bx+c=0 \quad \text{where } a\neq 0$$

is called Quadratic Equation in variable x.

3 Methods.

To solve Quadratic Equation there are three different methods named as;

1. Factorization method.
2. Completing Square method.
3. Quadratic Formula method.

Example 1

Solve by Factorization $x^2-7x+10=0$

$$x^2-2x-5x+10=0$$
$$x(x-2)-5(x-2)=0$$
$$(x-2)(x-5)=0$$

$10x^2$ → $-2x$, $-5x$

$x-2=0$, $x-5=0$

$\Rightarrow x=2$, $\Rightarrow x=5$ $\quad \{2, 5\}$

Example 2

Solve $x^2+4x-437=0$ by Completing Sq.

$$x^2+4x=437$$

Adding $(4/2)^2=(2)^2$ on both sides.

$$x^2+4x+(2)^2=437+(2)^2$$
$$(x+2)^2=437+4$$
$$(x+2)^2=441$$
$$x+2=\pm 21 \qquad \because \sqrt{441}=21$$

$x+2=21$, $x+2=-21$

$x=21-2$, $x=-21-2$

$x=19$, $x=-23$

$\{19, -23\}$

Example 3

Solve $6x^2+x-15=0$ by Q. Formula

Compairing $6x^2+x-15=0$ with $ax^2+bx+c=0$

we have $a=6$, $b=1$, $c=-15$

By using Quadratic Formula.

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$x=\frac{-1\pm\sqrt{(1)^2-4(6)(-15)}}{2(6)}$$

$$x=\frac{-1\pm\sqrt{1+360}}{12} \Rightarrow x=\frac{-1\pm\sqrt{361}}{12}$$

$$x=\frac{-1\pm 19}{12} \qquad \because \sqrt{361}=19$$

$$x=\frac{-1+19}{12} \ , \quad x=\frac{-1-19}{12}$$

$$x=\frac{18}{12} \ , \quad x=\frac{-20}{12}$$

$x=3/2$, $x=-5/3$ $\quad \left\{\frac{3}{2}, -\frac{5}{3}\right\}$

Example 4

Solve $8x^2-14x-15=0$ by Quadratic For

Compairing $8x^2-14x-15=0$ with $ax^2+bx+c=0$

We have $a=8$, $b=-14$, $c=-15$

By using $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

$$x=\frac{-(-14)\pm\sqrt{(-14)^2-4(8)(-15)}}{2(8)}$$

$$x=\frac{14\pm\sqrt{196+480}}{16} \Rightarrow x=\frac{14\pm\sqrt{676}}{16}$$

$$x=\frac{14\pm 26}{16} \qquad \because \sqrt{676}=26$$

$$x=\frac{14+26}{16} \ , \quad x=\frac{14-26}{16}$$

$$x=\frac{40}{16} \ , \quad x=\frac{-12}{16}$$

$x=5/2$, $x=-3/4$ $\quad \{5/2, -3/4$

Q Derive the Quadratic Formula.

Quadratic Equation in standard form is $ax^2+bx+c=0$

Dividing by a

$$\frac{ax^2}{a}+\frac{bx}{a}+\frac{c}{a}=\frac{0}{a}$$

$$x^2+\frac{b}{a}x+\frac{c}{a}=0$$

Now $$x^2+\frac{b}{a}x=-\frac{c}{a}$$

Adding $\left(\frac{1}{2}\cdot\frac{b}{a}\right)^2$ on both sides

$$x^2+\frac{b}{a}x+\left(\frac{1}{2}\frac{b}{a}\right)^2=\left(\frac{1}{2}\frac{b}{a}\right)^2-\frac{c}{a}$$

$$x^2+x\cdot\frac{b}{a}+\left(\frac{b}{2a}\right)^2=\frac{b^2}{4a^2}-\frac{c}{a}$$

$$(x)^2+2\cdot(x)\left(\frac{b}{2a}\right)+\left(\frac{b}{2a}\right)^2=\frac{b^2-4ac}{4a^2}$$

As $y^2+2yz+z^2=(y+z)^2$

So

$$\left(x+\frac{b}{2a}\right)^2=\frac{b^2-4ac}{4a^2}$$

$$x+\frac{b}{2a}=\pm\sqrt{\frac{b^2-4ac}{4a^2}}$$

$$x=-\frac{b}{2a}\pm\frac{\sqrt{b^2-4ac}}{2a}$$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

This is called Quadratic Formula.

Note

In Quadratic Equation $ax^2+bx+c=0$

1. a, b and c are real numbers.
2. The answer of Quadratic Equation are called its Roots.
3. Another name of Quadratic Equation is Second Degree Polynomial.

EXERCISE 4.1

Solve by FACTORIZATION.

Q.1 $3x^2+4x+1=0$

$3x^2+3x+x+1=0$

$3x(x+1)+1(x+1)=0$

$(x+1)(3x+1)=0$

$x+1=0$, $3x+1=0$

$x=-1$, $3x=-1$

$x=-1/3$

$3x^2$: $3x$, x

$\{-1, -\frac{1}{3}\}$

Q.2 $x^2+7x+12=0$

$x^2+4x+3x+12=0$

$x(x+4)+3(x+4)=0$

$(x+3)(x+4)=0$

$x+3=0$, $x+4=0$

$x=-3$, $x=-4$

$12x^2$: $4x$, $3x$

$\{-3, -4\}$

Q.3 $9x^2-12x-5=0$

$9x^2+3x-15x-5=0$

$3x(3x+1)-5(3x+1)=0$

$(3x+1)(3x-5)=0$

$3x+1=0$, $3x-5=0$

$3x=-1$, $3x=5$

$x=-1/3$, $x=5/3$

$-45x^2$: $3x$, $-15x$

$\{-\frac{1}{3}, \frac{5}{3}\}$

Q.4 $x^2-x=2$

$x^2-x-2=0$

$x^2+x-2x-2=0$

$x(x+1)-2(x+1)=0$

$(x-2)(x+1)=0$

$x-2=0$, $x+1=0$

$-2x^2$: x, $-2x$

$\Rightarrow x=2, x=-1$

$\{2, -1\}$

Q.5 $x(x+7)=(2x-1)(x+4)$

$x^2+7x=2x^2+8x-x-4$

$x^2+7x=2x^2+7x-4$

$2x^2-x^2+7x-7x-4=0$

$x^2-4=0$

$x^2-(2)^2=0$

$(x+2)(x-2)=0$

$x+2=0$, $x-2=0$

$x=-2$, $x=2$

$\{2, -2\}$

6

Q6 $\frac{x}{x+1} + \frac{x+1}{x} = \frac{5}{2}$

Multiplying by $2x(x+1)$

$2x(x+1)\cdot\frac{x}{x+1} + 2x(x+1)\cdot\frac{x+1}{x} = 2x(x+1)\frac{5}{2}$

$2x^2 + 2(x+1)(x+1) = 5x(x+1)$

$2x^2 + 2(x^2+2x+1) = 5x^2+5x$

$2x^2 + 2x^2 + 4x + 2 = 5x^2 + 5x$

$4x^2 + 4x + 2 = 5x^2 + 5x$

$5x^2 - 4x^2 + 5x - 4x - 2 = 0$

$x^2 + x - 2 = 0$

$x^2 - x + 2x - 2 = 0$ ($-2x^2$: $-x$, $2x$)

$x(x-1) + 2(x-1) = 0$

$(x-1)(x+2) = 0$

$x-1 = 0$, $x+2 = 0$

$\Rightarrow x = 1$, $x = -2$ $\{1, -2\}$

Q.7 $\frac{1}{x+1} + \frac{2}{x+2} = \frac{7}{x+5}$

Multiplying by $(x+1)(x+2)(x+5)$

$(x+1)(x+2)(x+5)\frac{1}{x+1} + (x+1)(x+2)(x+5)\frac{2}{x+2}$

$= (x+1)(x+2)(x+5)\frac{7}{x+5}$

$(x+2)(x+5) + 2(x+1)(x+5) = 7(x+1)(x+2)$

$x^2+5x+2x+10+2(x^2+5x+x+5) = 7(x^2+2x+x+2)$

$x^2 + 7x + 10 + 2x^2 + 12x + 10 = 7x^2 + 21x + 14$

$3x^2 + 19x + 20 = 7x^2 + 21x + 14$

$7x^2 - 3x^2 + 21x - 19x + 14 - 20 = 0$

$4x^2 + 2x - 6 = 0$

$2x^2 + x - 3 = 0$

$2x^2 - 2x + 3x - 3 = 0$ ($-6x^2$: $-2x$, $3x$)

$2x(x-1) + 3(x-1) = 0$

$(x-1)(2x+3) = 0$

$x-1 = 0$, $2x+3 = 0$

$x = 1$, $2x = -3$

$x = -3/2$ $\{1, -3/2\}$

Q.8 $\frac{a}{ax-1} + \frac{b}{bx-1} = a+b$

$\frac{a}{ax-1} - b + \frac{b}{bx-1} - a = 0$

$\frac{a-b(ax-1)}{ax-1} + \frac{b-a(bx-1)}{bx-1} = 0$

$\frac{a-abx+b}{ax-1} + \frac{b-abx+a}{bx-1} = 0$

$\frac{a+b-abx}{ax-1} + \frac{a+b-abx}{bx-1} = 0$

$(a+b-abx)\left\{\frac{1}{ax-1} + \frac{1}{bx-1}\right\} = 0$

$(a+b-abx)\left\{\frac{bx-1+ax-1}{(ax-1)(bx-1)}\right\} = 0$

$(a+b-abx)(ax+bx-2) = 0(ax-1)(bx-1)$

$(a+b-abx)(ax+bx-2) = 0$

Either $a+b-abx = 0$ or $ax+bx-2 = 0$

$\Rightarrow abx = a+b$, $(a+b)x = 2$

$\Rightarrow x = \frac{a+b}{ab}$, $x = \frac{2}{a+b}$

$\left\{\frac{a+b}{ab}, \frac{2}{a+b}\right\}$

* Solve By Completing Square.

Q.9 $x^2 - 2x - 899 = 0$

$x^2 - 2x = 899$

Adding $(\frac{2}{2})^2 = (1)^2$ on both sides

$x^2 - 2x + (-1)^2 = 899 + (-1)^2$

$(x-1)^2 = 899 + 1$

$(x-1)^2 = 900$

$\Rightarrow x - 1 = \pm 30$

$x - 1 = 30$, $x - 1 = -30$

$x = 30+1$, $x = -30+1$

$x = 31$, $x = -29$ $\{31, -29\}$

Q.10 $x^2 + 4x - 1085 = 0$

$x^2 + 4x = 1085$

Adding $(4/2)^2 = (2)^2$ on both sides

$x^2 + 4x + (2)^2 = 1085 + (2)^2$

$(x+2)^2 = 1085+4$

$(x+2)^2 = 1089$

$\Rightarrow x+2 = \pm 33.$

$x+2 = 33, \quad x+2 = -33$

$x = 33-2, \quad x = -33-2$

$x = 31, \quad x = -35$

$\{31, -35\}$

Q.11 $x^2+6x-567 = 0$

$x^2+6x = 567$

Adding $(\frac{6}{2})^2 = (3)^2$ on both sides

$x^2+6x+(3)^2 = 567+(3)^2$

$(x+3)^2 = 567+9$

$(x+3)^2 = 576$

$x+3 = \pm 24$

$x+3 = 24, \quad x+3 = -24$

$x = 24-3, \quad x = -24-3$

$x = 21, \quad x = -27$

$\{21, -27\}$

Q.12 $x^2-3x-648 = 0$

$x^2-3x = 648$

Adding $(\frac{3}{2})^2$ on both sides

$x^2-3x+(\frac{3}{2})^2 = 648+(\frac{3}{2})^2$

$(x-\frac{3}{2})^2 = 648+\frac{9}{4}$

$(x-\frac{3}{2})^2 = \frac{2592+9}{4}$

$(x-\frac{3}{2})^2 = \frac{2601}{4}$

$\Rightarrow x-\frac{3}{2} = \pm \frac{51}{2}$

$x-\frac{3}{2} = \frac{51}{2}, \quad x-\frac{3}{2} = -\frac{51}{2}$

$x = \frac{51}{2}+\frac{3}{2}, \quad x = -\frac{51}{2}+\frac{3}{2}$

$x = \frac{51+3}{2}, \quad x = \frac{-51+3}{2}$

$x = \frac{54}{2}, \quad x = \frac{-48}{2}$

$x = 27, \quad x = -24$ $\{27, -24\}$

Q.13 $x^2-x-1806 = 0$

$x^2-x = 1806$

Adding $(\frac{1}{2})^2$ on both sides

$x^2-x+(\frac{1}{2})^2 = 1806+(\frac{1}{2})^2$

$(x-\frac{1}{2})^2 = 1806+\frac{1}{4}$

$(x-\frac{1}{2})^2 = \frac{7224+1}{4}$

$(x-\frac{1}{2})^2 = \frac{7225}{4}$

$\Rightarrow x-\frac{1}{2} = \pm \frac{85}{2}$

$x-\frac{1}{2} = \frac{85}{2}, \quad x-\frac{1}{2} = -\frac{85}{2}$

$x = \frac{85}{2}+\frac{1}{2}, \quad x = -\frac{85}{2}+\frac{1}{2}$

$x = \frac{85+1}{2}, \quad x = \frac{-85+1}{2}$

$x = \frac{86}{2}, \quad x = \frac{-84}{2}$

$x = 43, \quad x = -42$ $\{43, -42\}$

Q.14 $2x^2+12x-110 = 0$

Dividing by 2. $x^2+6x-55 = 0$

$x^2+6x = 55$

Adding $(\frac{6}{2})^2 = (3)^2$ on both sides

$x^2+6x+(3)^2 = 55+(3)^2$

$(x+3)^2 = 55+9$

$(x+3)^2 = 64$

$x+3 = \pm 8$

$x+3 = 8, \quad x+3 = -8$

$x = 8-3, \quad x = -8-3$

$x = 5, \quad x = -11$ $\{5, -11\}$

★ Find roots by using Q. Formula.

Q.15 $5x^2-13x+6 = 0$

Compairing $ax^2+bx+c = 0$

we have $a = 5, b = -13, c = 6$

Using $x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$x = \frac{-(-13) \pm \sqrt{(-13)^2-4(5)(6)}}{2(5)}$

$x = \frac{13 \pm \sqrt{169-120}}{10}$

$x = \frac{13 \pm \sqrt{49}}{10} \Rightarrow x = \frac{13 \pm 7}{10}$

$x = \frac{13+7}{10}$, $x = \frac{13-7}{10}$

$x = \frac{20}{10}$, $x = \frac{6}{10}$

$x = 2$, $x = \frac{3}{5}$ $\left\{2, \frac{3}{5}\right\}$

Q.16 $4x^2 + 7x - 1 = 0$

Compairing $ax^2 + bx + c = 0$

we get $a = 4, b = 7, c = -1$

Using $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \frac{-7 \pm \sqrt{(7)^2 - 4(4)(-1)}}{2(4)}$

$x = \frac{-7 \pm \sqrt{49 + 16}}{8}$

$x = \frac{-7 \pm \sqrt{65}}{8}$, $\left\{\frac{-7 \pm \sqrt{65}}{8}\right\}$

Q.17 $15x^2 + 2ax - a^2 = 0$

Compairing $ax^2 + bx + c = 0$

$a = 15, b = 2a, c = -a^2$

Using $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \frac{-2a \pm \sqrt{(2a)^2 - 4(15)(-a^2)}}{2(15)}$

$x = \frac{-2a \pm \sqrt{4a^2 + 60a^2}}{30}$

$x = \frac{-2a \pm \sqrt{64a^2}}{30} \Rightarrow x = \frac{-2a \pm 8a}{30}$

$x = \frac{-2a + 8a}{30}$, $x = \frac{-2a - 8a}{30}$

$x = \frac{6a}{30}$, $x = \frac{-10a}{30}$

$x = a/5$, $x = -a/3$ $\left\{a/5, -a/3\right\}$

Q.18 $16x^2 + 8x + 1 = 0$

Compairing $ax^2 + bx + c = 0$.

we get $a = 16, b = 8, c = 1$

Using. $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \frac{-8 \pm \sqrt{(8)^2 - 4(16)(1)}}{2(16)}$

$x = \frac{-8 \pm \sqrt{64 - 64}}{32} \Rightarrow x = \frac{-8 \pm \sqrt{0}}{32}$

$x = \frac{-8}{32} \Rightarrow x = -1/4$ $\left\{-1/4\right\}$

Q.19

$(x-a)(x-b) + (x-b)(x-c) + (x-c)(x-a) = 0$

Simplyfying

$x^2 - bx - ax + ab + x^2 - cx - bx + bc + x^2 - ax - cx + ac = 0$

$3x^2 - 2ax - 2bx - 2cx + ab + bc + ac = 0$

$3x^2 - 2(a+b+c)x + ab + bc + ac = 0$

Using $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \frac{-[-2(a+b+c)] \pm \sqrt{[-2(a+b+c)]^2 - 4(3)(ab+bc+ac)}}{2(3)}$

$x = \frac{2(a+b+c) \pm 2\sqrt{(a+b+c)^2 - 3(ab+bc+ac)}}{6}$

$x = \frac{(a+b+c) \pm \sqrt{a^2+b^2+c^2+2ab+2bc+2ca-3ab-3bc-3ca}}{3}$

$x = \frac{(a+b+c) \pm \sqrt{a^2+b^2+c^2-ab-bc-ca}}{3}$

$\left\{\frac{(a+b+c) \pm \sqrt{a^2+b^2+c^2-ab-bc-ca}}{3}\right\}$

Q.20 $(a+b)x^2 + (a+2b+c)x + b+c = 0$

Using $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \frac{-(a+2b+c) \pm \sqrt{(a+2b+c)^2 - 4(a+b)(b+c)}}{2(a+b)}$

$x = \frac{-(a+2b+c) \pm \sqrt{(a+2b+c)^2 - 4(ab+ac+b^2+bc)}}{2(a+b)}$

$x = \frac{-(a+2b+c) \pm \sqrt{a^2+4b^2+c^2+4ab+4bc+2ac-4ab-4ac-4b^2-4bc}}{2(a+b)}$

$x = \frac{-(a+2b+c) \pm \sqrt{a^2+c^2-2ac}}{2(a+b)}$

$$x = \frac{-(a+2b+c) \pm \sqrt{(a-c)^2}}{2(a+b)}$$

$$x = \frac{-(a+2b+c) \pm (a-c)}{2(a+b)}$$

$$x = \frac{-(a+2b+c)+a-c}{2(a+b)} \quad , \quad x = \frac{-(a+2b+c)-a+c}{2(a+b)}$$

$$x = \frac{-a-2b-c+a-c}{2(a+b)} \quad , \quad x = \frac{-a-2b-c-a+c}{2(a+b)}$$

$$x = \frac{-2b-2c}{2(a+b)} \quad , \quad x = \frac{-2b-2a}{2(a+b)}$$

$$x = \frac{-2(b+c)}{2(a+b)} \quad , \quad x = \frac{-2(a+b)}{2(a+b)}$$

$$x = -\frac{(b+c)}{a+b} \quad , \quad x = -1$$

$$\left\{ -\frac{(b+c)}{a+b} , -1 \right\}$$

i)

Put $x^{1/4} = y$ then

$$y^2 - y - 6 = 0$$

Factorizing:

$-6y^2$ → $2y$, $-3y$

$$y^2 + 2y - 3y - 6 = 0$$

$$y(y+2) - 3(y+2) = 0$$

$$(y+2)(y-3) = 0$$

$$y+2 = 0 \; , \; y-3 = 0$$

$$y = -2 \; , \; y = 3$$

If $y = -2$, If $y = 3$

Then $x^{1/4} = -2$ Then $x^{1/4} = 3$

$$(x^{1/4})^4 = (-2)^4 \; , \; (x^{1/4})^4 = (3)^4$$

$$x = 16 \; , \; x = 81$$

$$\{16, 81\}$$

Type No 2

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