Exercise 2.4 (Solutions)

Question # 1(i)

Conditional: $\sim p \to q$

Converse: $q \to \sim p$

Inverse: $p \to \sim q$

Contrapositive: $\sim q \to p$

Question # 1(ii)

Conditional: $q \to p$

Converse: $p \to q$

Inverse: $\sim q \to \sim p$

Contrapositive: $\sim p \to \sim q$

Question # 1(iii)

Conditional: $\sim p \to \sim q$

Converse: $\sim q \to \sim p$

Inverse: $p \to q$

Contrapositive: $q \to p$

Question # 1(iv)

Do yourself as above

Question # 2 (i)

Statement: $(p \to \sim p) \vee (p \to q)$

p	q	$\sim p$	$p \to \sim p$	$p \to q$	$(p \to \sim p) \vee (p \to q)$
T	T	F	F	T	T
T	F	F	F	F	F
F	T	T	T	T	T
F	F	T	T	T	T

Question # 2 (ii)

Statement: $(p \wedge \sim p) \to q$

p	q	$\sim p$	$p \wedge \sim p$	$(p \wedge \sim p) \to q$
T	T	F	F	T
T	F	F	F	T
F	T	T	F	T
F	F	T	F	T

Question # 2 (iii)

Statement: $\sim (p \to q) \leftrightarrow (p \wedge \sim q)$

p	Q	$\sim q$	$p \to q$	$\sim (p \to q)$	$p \wedge \sim q$	$(p \wedge \sim q) \leftrightarrow \sim (p \to q)$
T	T	F	T	F	F	T
T	F	T	F	T	T	T
F	T	F	T	F	F	T
F	F	T	T	F	F	T

Tautology:

The statement which is true for all possible values of the variables in it is called *tautology*.

Contingency:

The statement which is true or false depending upon the truth values of the variables involved in it is called a *contingency*.

Absurdity or Contradiction:

The statement which is false for all the possible values of the variables involved in it is called an *absurdity* or *contradiction*.

Question # 3 (i)

Statement: $(p \wedge q) \rightarrow p$

P	q	$p \wedge q$	$p \wedge q \rightarrow p$
T	T	T	T
T	F	F	T
F	T	F	T
F	F	F	T

The last column of the above table shows that the statement is true for all values of p and q thus given statement is tautology.

Question # 3 (ii)

Statement: $p \rightarrow (p \vee q)$

p	q	$p \vee q$	$p \rightarrow (p \vee q)$
T	T	T	T
T	F	T	T
F	T	T	T
F	F	F	T

The last column of the above table shows that the statement is true for all values of p and q thus given statement is tautology

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Question # 3 (iii)

Statement: $\sim (p \rightarrow q) \rightarrow p$

p	Q	$p \rightarrow q$	$\sim (p \rightarrow q)$	$\sim (p \rightarrow q) \rightarrow p$
T	T	T	F	T
T	F	F	T	T
F	T	T	F	T
F	F	T	F	T

The last column of the above table shows that the statement is true for all values of p and q thus given statement is tautology.

Question # 3 (iv)

Statement: $\sim q \wedge (p \rightarrow q) \rightarrow \sim p$

p	q	$\sim p$	$\sim q$	$p \rightarrow q$	$\sim q \wedge (p \rightarrow q)$	$\sim q \wedge (p \rightarrow q) \rightarrow \sim p$
T	T	F	F	T	F	T
T	F	F	T	F	F	T
F	T	T	F	T	F	T
F	F	T	T	T	T	T

The last column of the above table shows that the statement is true for all values of p and q thus given statement is tautology.

Question # 4 (i)

Statement: $\sim (p \to q) \to p$

p	$\sim p$	$p \wedge \sim p$
T	F	F
F	T	F

The last column of the above table shows that the statement is false for all values of p and q thus given statement is absurdity.

Question # 4 (ii)

Statement: $p \to (q \to p)$

p	q	$q \to p$	$p \to (q \to p)$
T	T	T	T
T	F	T	T
F	T	F	T
F	F	T	T

The last column of the above table shows that the statement is true for all values of p and q thus given statement is tautology.

Question # 4 (iii)

Statement: $q \vee (\sim q \vee p)$

P	q	$\sim q$	$\sim q \vee p$	$q \vee (\sim q \vee p)$
T	T	F	T	T
T	F	T	T	T
F	T	F	F	T
F	F	T	T	T

The last column of the above table shows that the statement is true for all values of p and q thus given statement is tautology.

Question # 5

Consider the truth table

p	q	$\sim p$	$\sim q$	$p \wedge q$	$\sim p \wedge \sim q$	$p \vee (\sim p \wedge \sim q) \vee (p \wedge q)$	$p \vee (\sim p \wedge \sim q)$
T	T	F	F	T	F	T	T
T	F	F	T	F	F	T	T
F	T	T	F	F	F	F	F
F	F	T	T	F	T	T	T

The last two column of the above table are identical this shows that the statement $p \vee (\sim p \wedge \sim q) \vee (p \wedge q)$ and $p \vee (\sim p \wedge \sim q)$ are equal

i.e. $p \vee (\sim p \wedge \sim q) \vee (p \wedge q) = p \vee (\sim p \wedge \sim q)$

Made by: ***Atiq ur Rehman*** *(atiq@mathcity.org), http://www.mathcity.org*