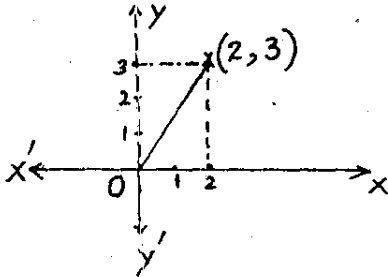
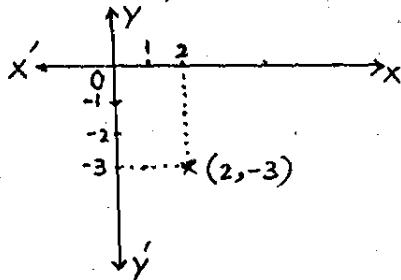


EXERCISE 1.3

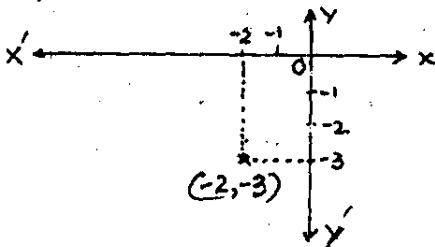
i) $2+3i = (2, 3)$



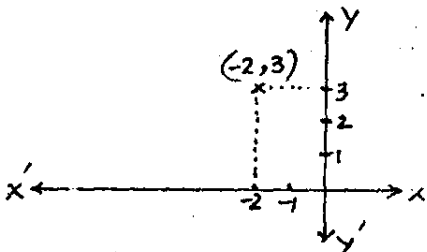
ii) $2-3i = (2, -3)$



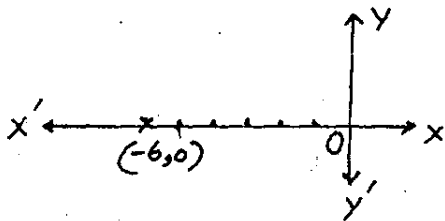
iii) $-2-3i = (-2, -3)$



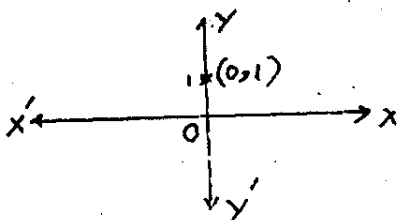
iv) $-2+3i = (-2, 3)$



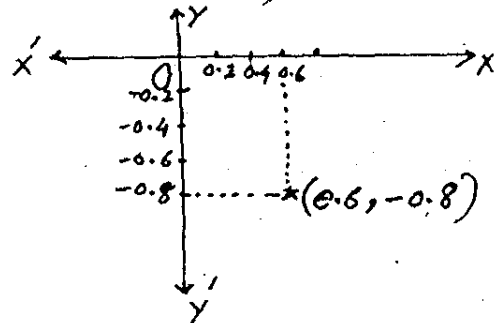
v) $-6 = -6 + i \cdot 0 = (-6, 0)$



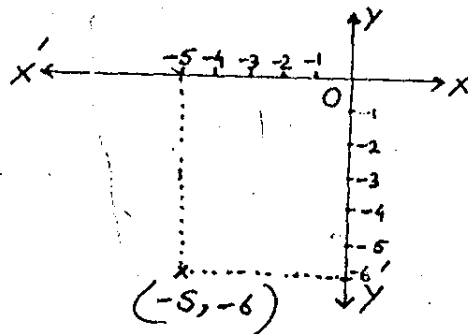
vi) $i = 0 + i = (0, 1)$



vii) $\frac{3}{5} - \frac{4}{5}i = 0.6 - 0.8i$
 $= (0.6, -0.8)$



viii) $-5-6i = (-5, -6)$



② i) let $z = -3i$

Multiplicative inverse of z .

$$= \frac{1}{z} = \frac{1}{-3i} = \frac{1}{-3i} \times \frac{i}{i} = \frac{i}{-3i^2}$$

$$= \frac{i}{-3(-1)} = \frac{i}{3} \text{ Ans.}$$

ii) let $z = 1-2i$

Multiplicative inverse of $z = \frac{1}{z}$

$$= \frac{1}{1-2i} = \frac{1}{1-2i} \times \frac{1+2i}{1+2i}$$

$$= \frac{1+2i}{(1)^2 - (2i)^2} = \frac{1+2i}{1-4i^2} = \frac{1+2i}{1-4(-1)}$$

$$= \frac{1+2i}{1+4} = \frac{1+2i}{5} = \frac{1}{5} + \frac{2}{5}i \text{ Ans.}$$

iii) let $z = -3-5i$

Multiplicative inverse of $z = \frac{1}{z}$

$$= \frac{1}{-3-5i} = \frac{1}{-3-5i} \times \frac{-3+5i}{-3+5i}$$

$$= \frac{-3+5i}{(-3)^2 - (5i)^2}$$

$$\frac{-3+5i}{9-25i^2} = \frac{-3+5i}{9+25}$$

$$= \frac{-3+5i}{34} = \frac{-3}{34} + \frac{5i}{34} \text{ Ans.}$$

iv) let $z = (1, 2) = 1+2i$

Multiplicative inverse of $z = \frac{1}{z}$

$$= \frac{1}{1+2i} = \frac{1}{1+2i} \times \frac{1-2i}{1-2i} = \frac{1-2i}{(1)^2 - (2i)^2}$$

$$= \frac{1-2i}{1-4i^2} = \frac{1-2i}{1-4(-1)} = \frac{1-2i}{1+4} = \frac{1-2i}{5}$$

$$= \frac{1}{5} - \frac{2}{5}i = \left(\frac{1}{5}, -\frac{2}{5}\right) \text{ Ans.}$$

③ i) $i^{101} = (i^2)^{50} \cdot i = (-1)^{50} \cdot i = 1 \cdot i = i$
 $= i$ Ans.

ii) $(-ai)^4 = (-a)^4 i^4 = a^4 (i^2)^2$
 $= a^4 (-1)^2 = a^4 \cdot 1 = a^4$ Ans.

iii) $i^{-3} = \frac{1}{i^3} = \frac{1}{i^2 \cdot i} = \frac{1}{(-1)(i)} = \frac{1}{-i}$
 $= \frac{1}{-i} \times \frac{i}{i} = \frac{i}{-i^2} = \frac{i}{-(-1)} = \frac{i}{1} = i$
 $= i$ Ans.

iv) $i^{-10} = \frac{1}{i^{10}} = \frac{1}{(i^2)^5} = \frac{1}{(-1)^5} = \frac{1}{-1} = -1$
 $= -1$ Ans.

④ Let $z = a+bi, a, b \in \mathbb{R}$
 Suppose that $\bar{z} = z$ (1)
 we shall prove that z is real
 $\therefore \bar{z} = z$

$$\Rightarrow a+ib = a+ib$$

$$\Rightarrow a-ib = a+ib$$

$$\Rightarrow a-ib - a - ib = 0$$

$$\Rightarrow -2ib = 0$$

$$\Rightarrow b = 0 \quad \because -2i \neq 0$$

\therefore (1) becomes

$$z = a + i(0) = a + 0 = a$$

$\Rightarrow z$ is real.

Conversely suppose that z is real.

Then we shall prove that

$$\bar{z} = z$$

$\because z$ is real

\Rightarrow Imaginary part of $z = 0$

$$\Rightarrow b = 0$$

\therefore (1) becomes

$$z = a + i(0) = a + 0 = a$$

$$\Rightarrow z = a \quad \text{--- (2)}$$

Taking conjugate on both sides

$$\bar{z} = \bar{a} \Rightarrow \bar{z} = a \quad \text{--- (3) } \because a \in \mathbb{R}$$

\therefore From (2) and (3), we get

$$\bar{z} = z \quad (\text{proved})$$

⑤ i) $5+2\sqrt{-4} = 5+2\sqrt{(-1)(4)}$
 $= 5+2\sqrt{-1}\sqrt{4} = 5+2(i)(2) = 5+4i$
 Ans.

ii) $(2+\sqrt{-3})(3+\sqrt{-3})$
 $= (2+\sqrt{(-1)(3)})(3+\sqrt{(-1)(3)})$
 $= (2+i\sqrt{3})(3+i\sqrt{3})$
 $= 6 + i2\sqrt{3} + i3\sqrt{3} + i^2(\sqrt{3})^2$
 $= 6 + 5\sqrt{3}i + (-1)(3)$
 $= 6 + 5\sqrt{3}i - 3 = 3 + 5\sqrt{3}i$ Ans.

iii) $\frac{2}{15+\sqrt{-8}} = \frac{2}{15+\sqrt{(-1)(8)}} = \frac{2}{15+i\sqrt{8}}$
 $= \frac{2}{15+i(2\sqrt{2})}$ $\left\{ \begin{array}{l} \sqrt{8} = \sqrt{2 \times 2 \times 2} \\ = \sqrt{2^2 \times 2} \\ = 2\sqrt{2} \end{array} \right.$

$$\begin{aligned}
 &= \frac{2}{\sqrt{5} + 2\sqrt{2}i} \quad \text{[24]} \\
 &= \frac{2}{\sqrt{5} + 2\sqrt{2}i} \times \frac{\sqrt{5} - 2\sqrt{2}i}{\sqrt{5} - 2\sqrt{2}i} \\
 &= \frac{2(\sqrt{5} - 2\sqrt{2}i)}{(\sqrt{5})^2 - (2\sqrt{2}i)^2} = \frac{2(\sqrt{5} - 2\sqrt{2}i)}{5 - 4(2)i^2} \\
 &= \frac{2(\sqrt{5} - 2\sqrt{2}i)}{5 + 8} = \frac{2\sqrt{5} - 4\sqrt{2}i}{13} \\
 &= \frac{2\sqrt{5}}{13} - \frac{4\sqrt{2}}{13}i \quad \underline{\text{Ans.}}
 \end{aligned}$$

$$\begin{aligned}
 \text{iv) } \frac{3}{\sqrt{6} - \sqrt{-12}} &= \frac{3}{\sqrt{6} - \sqrt{(-1)(12)}} = \frac{3}{\sqrt{6} - \sqrt{-1}\sqrt{12}} \\
 &= \frac{3}{\sqrt{6} - i\sqrt{12}} = \frac{3}{\sqrt{6} - i(2\sqrt{3})} \quad \left. \begin{array}{l} \sqrt{12} = \sqrt{2 \times 2 \times 3} \\ = \sqrt{2^2 \times 3} \\ = 2\sqrt{3} \end{array} \right\} \\
 &= \frac{3}{\sqrt{6} - 2\sqrt{3}i} \times \frac{\sqrt{6} + 2\sqrt{3}i}{\sqrt{6} + 2\sqrt{3}i} \\
 &= \frac{3(\sqrt{6} + 2\sqrt{3}i)}{(\sqrt{6})^2 - (2\sqrt{3}i)^2} = \frac{3(\sqrt{6} + 2\sqrt{3}i)}{6 - 4(3)i^2} \\
 &= \frac{3(\sqrt{6} + 2\sqrt{3}i)}{6 + 12} = \frac{3(\sqrt{6} + 2\sqrt{3}i)}{18} \\
 &= \frac{\sqrt{6} + 2\sqrt{3}i}{6} = \frac{\sqrt{6}}{6} + \frac{2\sqrt{3}}{6}i \\
 &= \frac{\sqrt{6}}{6} + \frac{\sqrt{3}}{3}i = \frac{\sqrt{6}}{\sqrt{6} \times \sqrt{6}} + \frac{\sqrt{3}}{\sqrt{3} \times \sqrt{3}}i \\
 &= \frac{1}{\sqrt{6}} + \frac{1}{\sqrt{3}}i \quad \underline{\text{Ans.}}
 \end{aligned}$$

⑥) $z^2 + \bar{z}^2$ is a real number.

$$\begin{aligned}
 \text{Let } z &= a + ib, \quad a, b \in \mathbb{R} \\
 \text{then } \bar{z} &= a - ib \\
 z^2 + \bar{z}^2 &= (a + ib)^2 + (a - ib)^2 \\
 &= a^2 + i^2b^2 + 2abi + a^2 + i^2b^2 - 2abi \\
 &= 2a^2 + 2i^2b^2 \\
 &= 2a^2 + 2(-1)b^2 = 2a^2 - 2b^2 \\
 &\text{which is real.}
 \end{aligned}$$

ii) $z^2 - \bar{z}^2$ is Imaginary.

Let $z = a + ib$, $a, b \in \mathbb{R}$

then $\bar{z} = a - ib$

$$\begin{aligned}
 \text{Now } z^2 - \bar{z}^2 &= (a + ib)^2 - (a - ib)^2 \\
 &= (a^2 + i^2b^2 + 2abi) - (a^2 + i^2b^2 - 2abi) \\
 &= a^2 + i^2b^2 + 2abi - a^2 - i^2b^2 + 2abi \\
 &= 4abi \text{ which is imaginary} \\
 &\text{number.}
 \end{aligned}$$

⑦ Simplify

$$\begin{aligned}
 \text{i) } \left(-\frac{1}{2} + \frac{\sqrt{3}}{2}i\right)^3 &= \left(-\frac{1}{2} + \frac{\sqrt{3}}{2}i\right)^2 \left(-\frac{1}{2} + \frac{\sqrt{3}}{2}i\right) \\
 &= \left[\left(-\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}i\right)^2 + 2\left(-\frac{1}{2}\right)\left(\frac{\sqrt{3}}{2}i\right) \right] \left(-\frac{1}{2} + \frac{\sqrt{3}}{2}i\right) \\
 &= \left(\frac{1}{4} + \frac{3}{4}i^2 - \frac{\sqrt{3}}{2}i\right) \left(-\frac{1}{2} + \frac{\sqrt{3}}{2}i\right) \\
 &= \left(\frac{1}{4} - \frac{3}{4} - \frac{\sqrt{3}}{2}i\right) \left(-\frac{1}{2} + \frac{\sqrt{3}}{2}i\right) \quad \left\{ \begin{array}{l} \frac{1}{4} - \frac{3}{4} \\ = \frac{1-3}{4} \\ = -\frac{2}{4} = -\frac{1}{2} \end{array} \right. \\
 &= \left(-\frac{1}{2} - \frac{\sqrt{3}}{2}i\right) \left(-\frac{1}{2} + \frac{\sqrt{3}}{2}i\right) \\
 &= \left(-\frac{1}{2}\right)^2 - \left(\frac{\sqrt{3}}{2}i\right)^2 = \frac{1}{4} - \frac{3}{4}i^2 \\
 &= \frac{1}{4} + \frac{3}{4} = \frac{1+3}{4} = \frac{4}{4} = 1 \quad \underline{\text{Ans.}}
 \end{aligned}$$

ii) $\left(-\frac{1}{2} - \frac{\sqrt{3}}{2}i\right)^3$

$$\begin{aligned}
 &= \left(-\frac{1}{2} - \frac{\sqrt{3}}{2}i\right)^2 \left(-\frac{1}{2} - \frac{\sqrt{3}}{2}i\right) \\
 &= \left[\left(-\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}i\right)^2 - 2\left(-\frac{1}{2}\right)\left(\frac{\sqrt{3}}{2}i\right) \right] \left(-\frac{1}{2} - \frac{\sqrt{3}}{2}i\right) \\
 &= \left(\frac{1}{4} - \frac{3}{4} + \frac{\sqrt{3}}{2}i\right) \left(-\frac{1}{2} - \frac{\sqrt{3}}{2}i\right) \\
 &= \left(-\frac{1}{2} + \frac{\sqrt{3}}{2}i\right) \left(-\frac{1}{2} - \frac{\sqrt{3}}{2}i\right) \\
 &= \left(-\frac{1}{2}\right)^2 - \left(\frac{\sqrt{3}}{2}i\right)^2 \\
 &= \frac{1}{4} - \frac{3}{4}i^2 = \frac{1}{4} + \frac{3}{4} = \frac{1+3}{4} = \frac{4}{4} = 1
 \end{aligned}$$

Ans

$$\begin{aligned} \text{iii)} & \left(-\frac{1}{2} + \frac{\sqrt{3}}{2}i\right)^{-2} \left(-\frac{1}{2} - \frac{\sqrt{3}}{2}i\right)^{-2} \\ & = \frac{-\frac{1}{2} - \frac{\sqrt{3}}{2}i}{\left(-\frac{1}{2} + \frac{\sqrt{3}}{2}i\right)^2} \\ & = \frac{-\frac{1}{2} - \frac{\sqrt{3}}{2}i}{\left(-\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}i\right)^2 + 2\left(-\frac{1}{2}\right)\left(\frac{\sqrt{3}}{2}i\right)} \\ & = \frac{-\frac{1}{2} - \frac{\sqrt{3}}{2}i}{\frac{1}{4} - \frac{3}{4} - \frac{\sqrt{3}}{2}i} = \frac{-\frac{1}{2} - \frac{\sqrt{3}}{2}i}{-\frac{1}{2} - \frac{\sqrt{3}}{2}i} = 1 \end{aligned}$$

$$\begin{aligned} \text{iii)} & \left(-\frac{1}{2} - \frac{\sqrt{3}}{2}i\right)^{-2} \left(-\frac{1}{2} - \frac{\sqrt{3}}{2}i\right)^{-1} \\ & \quad \text{(according to book)} \\ & = \left(-\frac{1}{2} - \frac{\sqrt{3}}{2}i\right)^{-3} = \frac{1}{-\frac{1}{2} - \frac{\sqrt{3}}{2}i} \\ & = \frac{1}{-\frac{1}{2} - \frac{\sqrt{3}}{2}i} \times \frac{-\frac{1}{2} + \frac{\sqrt{3}}{2}i}{-\frac{1}{2} + \frac{\sqrt{3}}{2}i} \\ & = \frac{-\frac{1}{2} + \frac{\sqrt{3}}{2}i}{\left(-\frac{1}{2}\right)^2 - \left(\frac{\sqrt{3}}{2}i\right)^2} = \frac{-\frac{1}{2} + \frac{\sqrt{3}}{2}i}{\frac{1}{4} + \frac{3}{4}} \\ & = \frac{-\frac{1}{2} + \frac{\sqrt{3}}{2}i}{\frac{4}{4}} = -\frac{1}{2} + \frac{\sqrt{3}}{2}i \end{aligned}$$

$$\begin{aligned} \text{iv)} & (a+bi)^2 = a^2 + b^2i^2 + 2abi \\ & = a^2 + b^2(-1) + 2abi \\ & = a^2 - b^2 + 2abi \end{aligned}$$

$$\begin{aligned} \text{v)} & (a+bi)^{-2} = \frac{1}{(a+bi)^2} = \frac{1}{a^2 + b^2i^2 + 2abi} \\ & = \frac{1}{a^2 - b^2 + 2abi} \\ & = \frac{1}{(a^2 - b^2) + 2abi} \times \frac{(a^2 - b^2) - 2abi}{(a^2 - b^2) - 2abi} \end{aligned}$$

$$\begin{aligned} \text{v)} & (a^2 - b^2) - 2abi \\ & = \frac{(a^2 - b^2) - 2abi}{(a^2 - b^2)^2 - (2abi)^2} \\ & = \frac{(a^2 - b^2) - 2abi}{a^4 + b^4 - 2a^2b^2 - 4a^2b^2i^2} \\ & = \frac{(a^2 - b^2) - 2abi}{a^4 + b^4 - 2a^2b^2 + 4a^2b^2} \\ & = \frac{(a^2 - b^2) - 2abi}{a^4 + b^4 + 2a^2b^2} = \frac{(a^2 - b^2) - 2abi}{(a^2 + b^2)^2} \\ & = \frac{a^2 - b^2}{(a^2 + b^2)^2} - \frac{2abi}{(a^2 + b^2)^2} \end{aligned}$$

$$\begin{aligned} \text{vi)} & (a+bi)^3 = a^3 + (bi)^3 + 3(a)(bi)(a+bi) \\ & = a^3 + b^3i^3 + 3abi(a+bi) \\ & = a^3 + b^3(-i) + 3a^2bi + 3ab^2i^2 \\ & = a^3 - b^3i + 3a^2bi - 3ab^2 \\ & = a^3 - 3ab^2 + 3a^2bi - b^3i \\ & = a^3 - 3ab^2 + i(3a^2b - b^3) \end{aligned}$$

$$\begin{aligned} \text{vii)} & (a-bi)^3 \\ & = (a)^3 - (bi)^3 - 3(a)(bi)(a-bi) \\ & = a^3 - b^3i^3 - 3abi(a-bi) \\ & = a^3 - b^3(-i) - 3a^2bi + 3ab^2i^2 \\ & = a^3 + b^3i - 3a^2bi - 3ab^2 \\ & = a^3 - 3ab^2 - 3a^2bi + b^3i \\ & = a^3 - 3ab^2 - i(3a^2b - b^3) \end{aligned}$$

$$\begin{aligned} \text{viii)} & (3 - \sqrt{-4})^{-3} = \frac{1}{[3 - \sqrt{-4}]^3} = \frac{1}{[3 - \sqrt{(-1)(4)}]^3} \\ & = \frac{1}{(3 - \sqrt{-1} \cdot 2)^3} = \frac{1}{[3 - i(2)]^3} = \frac{1}{(3 - 2i)^3} \\ & = \frac{1}{(3)^3 - (2i)^3 - 3(3)(2i)(3 - 2i)} \\ & = \frac{1}{27 - 8i^3 - 18i(3 - 2i)} \end{aligned}$$

$$\begin{aligned}
 &= \frac{1}{27 - 8(-i) - 54i + 36i^2} \\
 &= \frac{1}{27 + 8i - 54i - 36} \\
 &= \frac{1}{-9 - 46i} \\
 &= \frac{1}{-9 - 46i} \times \frac{-9 + 46i}{-9 + 46i} \\
 &= \frac{-9 + 46i}{(-9)^2 - (46i)^2} = \frac{-9 + 46i}{81 - 2116i^2} \\
 &= \frac{-9 + 46i}{81 + 2116} = \frac{-9 + 46i}{2197} \\
 &= \frac{-9}{2197} + \frac{46i}{2197} \text{ Ans.}
 \end{aligned}$$

End of Chapter No. 1.

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